MATH327 – HOMEWORK SOLUTIONS

HOMEWORK #4

Section 1.5: Problems 1, 2, 3, 5
Section 2.1: Problems 1,4,7,9,10,14

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Problem 1.5.1.

a) Yes, it is reflexive, antisymmetric, and transitive and therefore it is a partial order on $\mathbb{R}$. It is also a total order.

b) No, as it is not reflexive. For example, it is not true that $2 < 2$.

c) $(\mathbb{R}, \preceq)$, where $a \preceq b$ means $a^2 \leq b^2$ is not a partial order as it is not antisymmetric. While $a \preceq b$ and $b \preceq a$ imply that $a^2 = b^2$, the latter does not mean that $a = b$.

d) This relation is not antisymmetric. Consider $(2, 3)$ and $(2, 1)$. Clearly $(2, 3) \preceq (2, 1)$ and $(2, 1) \preceq (2, 3)$ but $(2, 3) \neq (2, 1)$.

e) This is a partial order but not a total order. Consider, for example, $(2, 3)$ and $(3, 4)$. Neither $(2, 3) \preceq (3, 4)$, nor $(3, 4) \preceq (2, 3)$.

f) This relation is not antisymmetric. Consider two words $w_1 = cat$ and $w_2 = pit$ (in this example the alphabet is the standard English alphabet). They have the same length so $w_1 \preceq w_2$ and $w_2 \preceq w_1$. But they are not the same words.

Problem 1.5.2.

a) $\{1, 10, 100, 1000, 1001, 101, 1010, 11, 110, 111\}$

b) $\{1, 11, 111, 110, 10, 101, 1010, 100, 1001, 1000\}$

Problem 1.5.3.

a) $a \prec b \prec c \prec d$ (other follow by transitivity).

b) $a \prec b$ and $c \prec d$.

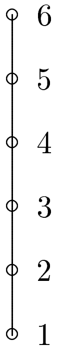
c) $a \prec b$, $a \prec d$, and $c \prec d$.

d) $a \prec b$, $a \prec c$, and $a \prec d$.

e) $a \prec d \prec e$, $b \prec e$, and $b \prec c \prec f$ (other follow by transitivity).

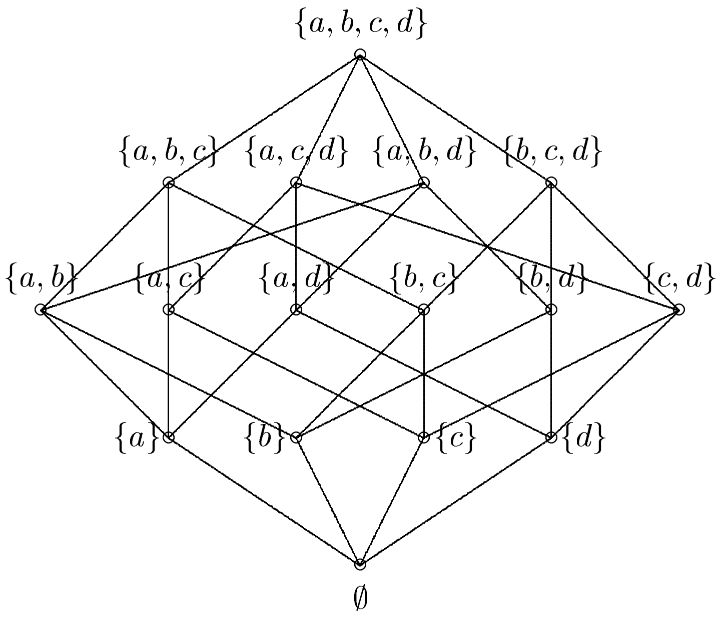
f) $a \prec f$, $g \prec f$, $d \prec b \prec c$, $d \prec h \prec i$, $\prec c$, $e \prec i$ (all the other relations follow by transitivity).

Problem 1.5.5.

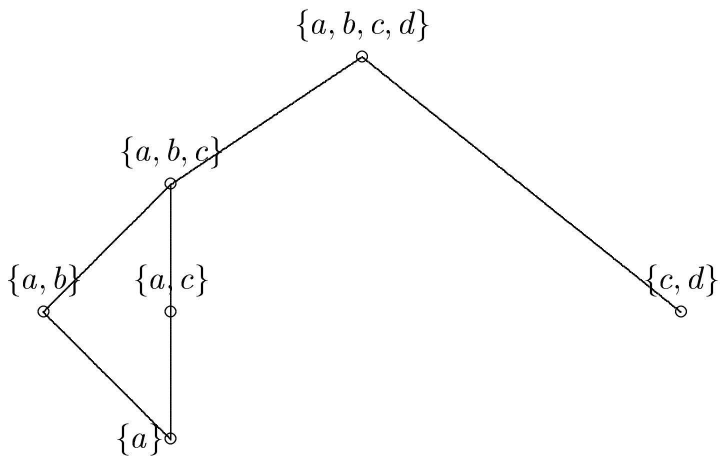


(a) The Hesse diagram for $(\{1,2,3,4,5,6\}, \leq)$

b)



(b') The Hesse diagram for $(\mathcal{P}(\{a,b,c,d\}), \subseteq)$



(b) The Hesse diagram for $(\{\{a\},\{a,b\},\{a,c\},\{c,d\},\{a,b,c\},\{a,b,c,d\}\},\subseteq)$

Problem 2.1.1

a) It is not a function as $f(3) = 1$ and $f(3) = 3$.

b) It is not a function with domain $\{1, 2, 3, 4\}$ as $f(3)$ is not defined.

c) Yes, it is a function.

d) It is not a function as $f(1) = 1$ and $f(1) = 2$.

e) Yes, it is a function.

Problem 2.1.4

a) $f(n) = 2n$.

b) $f(n) = 1+$ integer part of $n/100$.

c) $f(n) = n \text{mod} 3 + 1$.

d) $f(n) = n$.

Problem 2.1.7

$$\{(1,-3),(3,1),(5,5),(2,5),(4,17)\}$$

The function is not 1-1 as $f(5) = f(2)$.

Problem 2.1.9

a) Write $x^2 = |x|^2$ and the rest follows from the definition $\sqrt{x^2} = \sqrt{|x|^2} = |x|$ ($|x|$ is always non-negative).

b) As both sides of this inequality are non-negative integers we can square it to get

$$|x+y| \le |x| + |y| \Leftrightarrow (x+y)^2 \le (|x|+|y|)^2 \Leftrightarrow x^2 + y^2 + 2xy \le x^2 + y^2 + 2|x||y| \Leftrightarrow$$

$$\Leftrightarrow xy \le |xy|.$$

The last inequality is obvious (you can verify by checking four cases of different sign of x and y).

Problem 2.1.10

a) It is not 1-1 as $g(1) = g(-1)$. Neither it is onto as $g(x) \ge 1$.

b) It is not 1-1 as $g(1) = g(-1)$ but now it is clearly onto as any natural number can be written as $|x| + 1$. Simply take all positive integers and the zero for x.

Problem 2.1.14 Write $f(x) = (x+7)^2 - 100$.

a) 1-1 but not onto. $f(n)$ is strictly increasing for $n \ge -7$, hence it must be 1-1. Also, there is no natural number n such that $f(n) = -98$. This would mean that $f(n) = (n+7)^2 - 100 = -98$ or $(n+7)^2 = 2$ which has no solutions in natural numbers.

b) not 1-1 and not onto. To see this take $f(-17) = f(-3) = 0$, and observe that there is no integer such that $f(m) = -98$. This would mean that $f(m) = (m+7)^2 - 100 = -98$ or $(m+7)^2 = 2$ which has no integer solutions.

c) not 1-1 but onto. Just draw this parabola in the XY-plane.