MATH314 – HOMEWORK SOLUTIONS

HOMEWORK #8

Section 6.1: Problems 1(a)(g)(k),4,6,7,10,27

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Problem 6.1.1

a)

$$p(\lambda) = \det \begin{pmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{pmatrix} = (3-\lambda)(1-\lambda) - 8 = \lambda^2 - 4\lambda - 5 = (\lambda-5)(\lambda+1).$$

For $\lambda = 5$ we get

$$\begin{pmatrix} -2 & 2 \\ 4 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

and, hence,

$$E_{\lambda=5} = \text{span}\{(1,1)\}.$$

For $\lambda = -1$ we get

$$\begin{pmatrix} 4 & 2 \\ 4 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1/2 \\ 0 & 0 \end{pmatrix}$$

and, hence,

$$E_{\lambda=-1} = \text{span}\{(1,-2)\}.$$

g)

$$p(\lambda) = \det \begin{pmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^2(2-\lambda).$$

For $\lambda = 1$ we get

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and, hence,

$$E_{\lambda=1} = \{(t, -s, s)\} = \text{span}\{(1, 0, 0), (0. - 1, 1)\}.$$

For $\lambda = 2$ we get

$$\begin{pmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

and, hence,

$$E_{\lambda=2} = \text{span}\{(1, 1, 0)\}.$$

k)

$$p(\lambda) = \det \begin{pmatrix} 2-\lambda & 0 & 0 & 0 \\ 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 3-\lambda & 0 \\ 0 & 0 & 0 & 4-\lambda \end{pmatrix} = (2-\lambda)^2(3-\lambda)(4-\lambda).$$

For $\lambda = 2$ we get

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

and, hence,

$$E_{\lambda=2} = \{(t, s, 0, 0)\} = \text{span}\{(1, 0, 0, 0), (0, 1, 0, 0)\}.$$

For $\lambda = 3$ we get

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and, hence,

$$E_{\lambda=2} = \{(0, 0, t, 0)\} = \text{span}\{(0, 0, 1, 0)\}.$$

For $\lambda = 2$ we get

$$\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and, hence,

$$E_{\lambda=4} = \{(0, 0, 0, t)\} = \text{span}\{(0, 0, 0, 1)\}.$$

Problem 6.1.4,6,7,10,27 Solutions to all of these problems were handed out on April 25, 2002. You can also pick them up in a box outside my office door.