MATH314 – HOMEWORK SOLUTIONS

HOMEWORK #1

Section 1.1: Problems 1(b)(c), 3(a)(d), 5(c), 6(d), 9
Section 1.2: Problems 1, 2, 3(d), 5(c)(j), 10

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Problem 1.1.1

b) $x_3 = 3$, $x_2 = 1$, $x_1 = 8 - 3 - 1 = 4$.

c) $x_4 = 1$, $x_3 = 3$, $x_2 = 0$, $x_1 = 5 - 1 - 6 = -2$.

Problem 1.1.3

a) These are two intersecting lines, with the point of intersection $P = (3, 1)$.

d) These are three lines intersecting at 3 distinct points (a triangle) $P_1 = (1, 0), P_2 = (0, 1)$, and $P_3 = (3, 2)$. Hence, there are no solutions of the system as there is no common point to these three lines.

Problem 1.1.5

c) $2x_1 + x_2 + 4x_3 = -1, \quad 4x_1 - 2x_2 + 3x_3 = 4, \quad 5x_1 + 2x_2 + 6x_3 = -1.$

Problem 1.1.6

c)

$$\left(\begin{array}{cc|c} 4 & 3 & 4 \\ 2/3 & 4 & 3 \end{array}\right) \simeq \left(\begin{array}{cc|c} 4 & 3 & 4 \\ 0 & 7/2 & 7/3 \end{array}\right) \simeq \left(\begin{array}{cc|c} 4 & 3 & 4 \\ 0 & 1 & 2/3 \end{array}\right).$$

Hence, $x_2 = 2/3$ and, by back substitution, $x_1 = 1/2$. Or, reducing farther, we get

$$\left(\begin{array}{cc|c} 4 & 3 & 4 \\ 0 & 1 & 2/3 \end{array}\right) \simeq \left(\begin{array}{cc|c} 4 & 0 & 2 \\ 0 & 1 & 2/3 \end{array}\right) \simeq \left(\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & 2/3 \end{array}\right)$$

which yields the same result.

d)

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 3 \\ -1 & 2 & 3 & 7 \end{array}\right) \simeq \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -5 & 3 & 1 \\ 0 & 4 & 2 & 8 \end{array}\right) \simeq \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -1 & 5 & 9 \\ 0 & 4 & 2 & 8 \end{array}\right) \simeq$$

$$\simeq \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -5 & -9 \\ 0 & 1 & 1/2 & 2 \end{array}\right) \simeq \left(\begin{array}{ccc|c} 1 & 0 & 9 & 19 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 11/2 & 11 \end{array}\right) \simeq \left(\begin{array}{ccc|c} 1 & 0 & 9 & 19 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right) \simeq$$

$$\simeq \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right).$$

Problem 1.1.9

a) Note that, if $m_1 \neq m_2$ we can subtract the second equation from the first to get

$$(m_2 - m_1)x_1 = b_1 - b_2,$$

which gives

$$x_1 = \frac{b_1 - b_2}{m_2 - m_1}$$

and then x_1 is uniquely determined by the first (or second) equation.

b) If $m_1 = m_2$ subtracting as in part (a) gives $0 = b_1 - b_2$. This is consistent if and only if $b_1 = b_2$.

c) In part (a) m_i's are the slopes of the corresponding lines. These two lines intersect exactly at one point if and only if they have different slopes. In part (b) $b_1 = b_2$ means that both equations are the same and they describe exactly the same line. The system is consistent and it has infinitely many solutions.

Problem 1.2.1 (a) YES but not reduced, (b) NO, (c) YES and reduced, (d) YES and reduced, (e) NO, (f) NO, (g) YES and reduced, (h) YES but not reduced.

Problem 1.2.2

a) inconsistent,

b) consistent, unique solution $(x_1, x_2) = (4, -1)$,

c) consistent, no unique solution,

d) consistent, unique solution $(x_1, x_2, x_3) = (4, 5, 2)$.

f) inconsistent,

d) consistent, unique solution $(x_1, x_2, x_3) = (5, 3, 2)$.

Problem 1.2.3(d) We choose free parameters for unknown variables corresponding to the pivot-less columns. Here it is $x_2 = \alpha$, and $x_4 = \beta$. Then the two equations can be solved for the remaining two unknowns and

$$(x_1, x_2, x_3, x_4) = (5 - 2\alpha - \beta, \alpha, 4 - 3\beta, \beta).$$

Problem 1.2.5

c)

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 3 & -2 & 0 \end{array}\right) \simeq \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 0 \end{array}\right)$$

which shows that the system is consistent, however, it only has the trivial solution $(x_1, x_2) = (0, 0)$.

j)

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{array}\right) \simeq \left(\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \simeq \left(\begin{array}{cccc|c} 1 & 0 & -5 & 1 & -13 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \simeq$$

$$\simeq \left(\begin{array}{cccc|c} 1 & 0 & -5 & 1 & -13 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \simeq \left(\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 2 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right).$$

From this, be setting the unknown $x_4 = t$ (4th, pivot-less column), we immediately get a solution $(x_1, x_2, x_3, x_4) = (2 - 6t, 4 + t, 3 - t, t)$.

Problem 1.2.10 We first reduce the matrix to the row-echelon form

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right) \simeq \left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & a-3 & b-2 \end{array}\right) \simeq \left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & b-4 \end{array}\right).$$

If $a \neq 5$ then all rows have pivots and the system will have a unique solution. Hence,

a) If $a = 5$ and $b = 4$ then the third row is identically zero and, therefore ,the system is consistent and it has infinitely many solutions.

b) If $a = 5$ but $b \neq 4$ then the system is obviously inconsistent.