1. Find an orthonormal basis for the row space of $\mathbf{A}$ containing the first row.

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ -2 & -2 & 1 \\ 2 & 0 & -1 \end{pmatrix}.$$

2. Consider the following linear homogeneous equations

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 - x_2 + 2x_3 + 2x_4 &= 0 \\ x_2 + x_4 &= 0. \end{aligned}$$

a) Find all the solutions of the system and write it as a linear subspace W in $\mathbb{R}^4$.

b) The space $W \subset \mathbb{R}^4$ found in (a) is a linear subspace in $\mathbb{R}^4$. Find an orthonormal basis for W.

c) Find $W^\perp$ and write an orthonormal basis for it.

3. Given the following symmetric matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix},$$

find

a) the characteristic polynomial of $\mathbf{A}$,

b) all eigenvalues of $\mathbf{A}$,

c) all eigenvectors of $\mathbf{A}$,

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Find the associated matrix $\mathbf{A}$ of T. Compute $T\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.