MATH180 – HOMEWORK SOLUTIONS

HOMEWORK #8

Section 6.3: 5, 11

Section 6.4: 3, 7, 11, 15, 17-39 (odd only

Section 6.5: 1-27 (odd only), 33, 37, 45

EXERCISES 6.3, page 430

5. a. $A = 4$

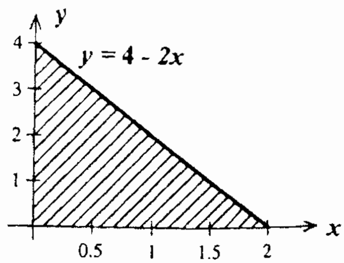
b. $\Delta x = \frac{2}{5} = 0.4$; $x_1 = 0, x_2 = 0.4, x_3 = 0.8, x_4 = 1.2$
$x_5 = 1.6,$

$$A \approx 0.4\{[4-2(0)] + [4-2(0.4)] + [4-2(0.8)] + [4-2(1.2)] + [4-2(1.6)]\} = 4.8$$

c. $\Delta x = \frac{2}{10} = 0.2, x_1 = 0, x_2 = 0.2, x_3 = 0.4, ..., x_{10} = 1.8.$

$$A \approx 0.2\{[4-2(0)] + [4-2(0.2)] + [4-2(0.4)] + \cdots + [4-2(1.8)]\} = 4.4$$

d. Yes.



11. a. $\Delta x = \frac{1}{2}$, $x_1 = 0, x_2 = \frac{1}{2}$. The Riemann sum is

$$f(x_1)\Delta x + f(x_2)\Delta x = \left[(0)^3 + (\tfrac{1}{2})^3\right]\tfrac{1}{2} = \tfrac{1}{16} = 0.0625.$$

b. $\Delta x = \frac{1}{5}, x_1 = 0, x_2 = \frac{1}{5}, x_3 = \frac{2}{5}, x_4 = \frac{3}{5}, x_5 = \frac{4}{5}$. The Riemann sum is $f(x_1)\Delta x + f(x_2)\Delta x + \cdots f(x_5)\Delta x = \left[(\tfrac{1}{5})^3 + (\tfrac{2}{5})^3 + \cdots + (\tfrac{4}{5})^3\right]\tfrac{1}{5} = \tfrac{100}{625} = 0.16.$

c. $\Delta x = \frac{1}{10}; x_1 = 0, x_2 = \frac{1}{10}, x_3 = \frac{2}{10}, \cdots, x_{10} = \frac{9}{10}.$

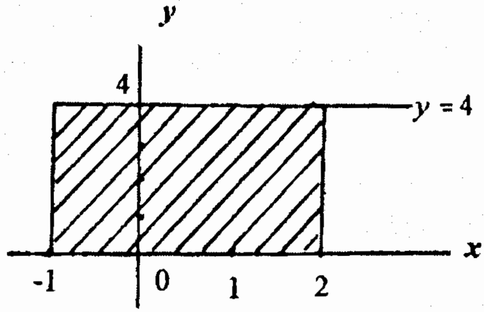
The Riemann sum is

$$f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_{10})\Delta x = \left[(\tfrac{1}{10})^3 + (\tfrac{2}{10})^3 + \cdots + (\tfrac{9}{10})^3\right]\tfrac{1}{10}$$
$$= \tfrac{2025}{10{,}000} = 0.2025 \approx 0.2 \text{ sq units.}$$

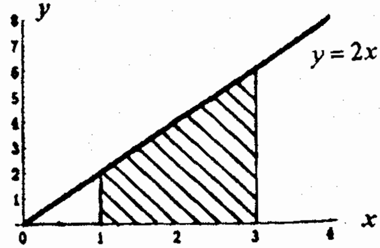
The Riemann sum seems to approach 0.2.

EXERCISES 6.4, page 439

2. $A = \int_{-1}^{2} 4\,dx = 4x\Big|_{-1}^{2} = 8-(-4) = 12,$ or 12 sq units. The region is a rectangle whose area is $4[2-(-1)] = 12$ sq units.



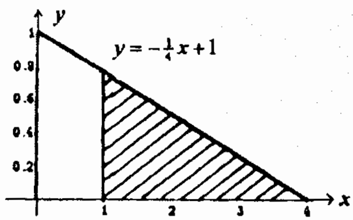
3. $A = \int_{1}^{3} 2x\,dx = x^2\Big|_{1}^{3} = 9-1 = 8,$ or 8 sq units. The region is a parallelogram of area $(1/2)(3-1)(2+6) = 8$ sq units.



4. $A = \int_{1}^{4}\left(-\frac{1}{4}x+1\right)dx = -\frac{1}{8}x^2 + x\Big|_{1}^{4}$

$= (-2+4)-(-\frac{1}{8}+1) = \frac{9}{8},$

or 9/8 sq units. The region is a triangle whose area is $(1/2)(3)(3/4) = 9/8$ sq units.



5. $A = \int_{-1}^{2}(2x+3)\,dx = x^2+3x\Big|_{-1}^{2} = (4+6)-(1-3) = 12$, or 12 sq. units.

6. $A = \int_{2}^{4}(4x-1)\,dx = 2x^2 - x\Big|_{2}^{4} = (32-4)-(8-2) = 22$, or 22 sq units.

7. $A = \int_{-1}^{2}(-x^2+4)\,dx = -\frac{1}{3}x^3 + 4x\Big|_{-1}^{2} = \left(-\frac{8}{3}+8\right)-\left(\frac{1}{3}-4\right) = 9$, or 9 sq units.

8. $A = \int_{0}^{4}(4x-x^2)dx = 2x^2 - \frac{1}{3}x^3\Big|_{0}^{4} = 32 - \frac{64}{3} = \frac{32}{3},$ or $\frac{32}{3}$ sq units.

9. $A = \int_{1}^{2}\frac{1}{x}dx = \ln|x|\Big|_{1}^{2} = \ln 2 - \ln 1 = \ln 2$, or ln 2 sq units.

10. $A = \int_2^4 \frac{1}{x^2}\,dx = \int_2^4 x^{-2}\,dx = -\frac{1}{x}\Big|_2^4 = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$, or $\frac{1}{4}$ sq units.

11. $A = \int_1^9 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_1^9 = \frac{2}{3}(27-1) = \frac{52}{3}$, or $17\frac{1}{3}$ sq units.

12. $A = \int_1^3 x^3\,dx = \frac{1}{4}x^4\Big|_1^3 = \frac{1}{4}(81-1) = 20$ sq units.

13. $A = \int_{-8}^{-1} (1 - x^{1/3})\,dx = x - \frac{3}{4}x^{4/3}\Big|_{-8}^{-1} = (-1-\frac{3}{4}) - (-8-12) = 18\frac{1}{4}$, or $18\frac{1}{4}$ sq units.

14. $A = \int_1^9 x^{-1/2}\,dx = 2x^{1/2}\Big|_1^9 = 2(3-1) = 4$, or 4 sq units.

15. $A = \int_0^2 e^x dx = e^x\Big|_0^2 = (e^2-1)$, or approximately 6.39 sq units.

16. $A = \int_1^2 (e^x - x)dx = e^x - \frac{1}{2}x^2\Big|_1^2 = (e^2-2) - (e-\frac{1}{2}) = (e^2 - e - \frac{3}{2})$ or approximately 3.17 sq units.

17. $\int_2^4 3\,dx = 3x\big|_2^4 = 3(4-2) = 6.$

18. $\int_{-1}^2 -2\,dx = -2x\Big|_{-1}^2 = -4-2 = -6.$

19. $\int_1^3 (2x+3)\,dx = x^2 + 3x\Big|_1^3 = (9+9) - (1+3) = 14.$

20. $\int_{-1}^0 (4-x)\,dx = 4x - \frac{1}{2}x^2\Big|_{-1}^0 = 0 - (-4-\frac{1}{2}) = 4\frac{1}{2}$

21. $\int_{-1}^3 2x^2\,dx = \frac{2}{3}x^3\Big|_{-1}^3 = \frac{2}{3}(27) - \frac{2}{3}(-1) = \frac{56}{3}.$

22. $\int_0^2 8x^3\,dx = 2x^4\Big|_0^2 = 32.$

23. $\int_{-2}^2 (x^2-1)\,dx = \frac{1}{3}x^3 - x\Big|_{-2}^2 = \left(\frac{8}{3}-2\right) - \left(-\frac{8}{3}+2\right) = \frac{4}{3}.$

24. $\int_1^4 \sqrt{u}\;du = \frac{2}{3}u^{3/2}\Big|_1^4 = \frac{2}{3}(8) - \frac{2}{3}(1) = \frac{14}{3}.$

25. $\int_1^8 4x^{1/3}\,dx = (4)(\frac{3}{4})x^{4/3}\Big|_1^8 = 3(16-1) = 45.$

26. $\int_1^4 2x^{-3/2}\,dx = (2)(-2)x^{-1/2}\Big|_1^4 = -4(\frac{1}{2}-1) - 2.$

27. $\int_0^1 (x^3 - 2x^2 + 1)\,dx = \frac{1}{4}x^4 - \frac{2}{3}x^3 + x\Big|_0^1 = \frac{1}{4} - \frac{2}{3} + 1 = \frac{7}{12}$

28. $\int_1^2 (t^5 - t^3 + 1)\,dt = \frac{1}{6}t^6 - \frac{1}{4}t^4 + t\Big|_1^2 = \left[\frac{1}{6}(64) - \frac{1}{4}(16) + 2\right] - [\frac{1}{6} - \frac{1}{4} + 1] = 7\frac{3}{4}.$

29. $\int_2^4 \frac{1}{x}\,dx = \ln|x|\Big|_2^4 = \ln 4 - \ln 2 = \ln(\frac{4}{2}) = \ln 2.$

30. $\int_1^3 \frac{2}{x}\,dx = 2\ln|x|\Big|_1^3 = 2\ln 3.$

31. $\int_0^4 x(x^2 - 1)\,dx = \int_0^4 (x^3 - x)\,dx = \frac{1}{4}x^4 - \frac{1}{2}x^2\Big|_0^4 = 64 - 8 = 56.$

32. $\int_0^2 (x-4)(x-1)\,dx = \int_0^2 (x^2 - 5x + 4)\,dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\Big|_0^2 = \frac{8}{3} - 10 + 8 = \frac{2}{3}.$

33. $\int_1^3 (t^2 - t)^2\,dt = \int_1^3 t^4 - 2t^3 + t^2)\,dt = \frac{1}{5}t^5 - \frac{1}{2}t^4 + \frac{1}{3}t^3\Big|_1^3$

$$= \left(\frac{243}{5} - \frac{81}{2} + \frac{27}{3}\right) - \left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right) = \frac{512}{30} = \frac{256}{15}.$$

34. $\int_{-1}^1 (x^2 - 1)^2\,dx = \int_{-1}^1 (x^4 - 2x^2 + 1)\,dx = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x\Big|_{-1}^1$

$$= \left(\frac{1}{5} - \frac{2}{3} + 1)\right) - \left(-\frac{1}{5} + \frac{2}{3} - 1\right) = \frac{16}{15}.$$

35. $\int_{-3}^{-1} x^{-2}\,dx = -\left.\frac{1}{x}\right|_{-3}^{-1} = 1-\frac{1}{3}=\frac{2}{3}.$ 36. $\int_1^2 2x^{-3}\,dx = -\left.\frac{1}{x^2}\right|_1^2 = -\frac{1}{4}+1=\frac{3}{4}.$

37. $\int_1^4\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx = \int_1^4 (x^{1/2}-x^{-1/2})\,dx = \left.\frac{2}{3}x^{3/2}-2x^{1/2}\right|_1^4$

$$= \left(\frac{16}{3}-4\right)-\left(\frac{2}{3}-2\right)=\frac{8}{3}.$$

38. $\int_0^1 \sqrt{2x}(\sqrt{x}+\sqrt{2})\,dx = \int_0^1 (\sqrt{2}x+2\sqrt{x})\,dx = \left.\frac{\sqrt{2}}{2}x^2+\frac{4}{3}x^{3/2}\right|_0^1 = \frac{\sqrt{2}}{2}+\frac{4}{3}.$

39. $\int_1^4 \frac{3x^3-2x^2+4}{x^2}\,dx = \int_1^4 (3x-2+4x^{-2})\,dx = \left.\frac{3}{2}x^2-2x-\frac{4}{x}\right|_1^4$

$$=(24-8-1)-(\tfrac{3}{2}-2-40=\tfrac{39}{2}.$$

EXERCISES 6.5, page 449

1. Let $u = x^2-1$ so that $du = 2x\,dx$ or $x\,dx = \frac{1}{2}\,du$. Also, if $x=0$, then $u=-1$ and if $x=2$, then $u=3$. So

$$\int_0^2 x(x^2-1)^3\,dx = \frac{1}{2}\int_{-1}^3 u^3\,du = \left.\tfrac{1}{8}u^4\right|_{-1}^3 = \tfrac{1}{8}(81)-\tfrac{1}{8}(1)=10.$$

2. Let $u = 2x^3-1$ so that $du = 6x^2\,dx$ or $x^2\,dx = \frac{1}{6}\,du$. Also, if $x=0$, $u=-1$, and if $x=1$, then $u=1$. So

$$\int_0^1 x^2(2x^3-1)^4\,dx = \tfrac{1}{6}\int_{-1}^1 u^4\,du = \left.\tfrac{1}{30}u^5\right|_{-1}^1 = \tfrac{1}{30}-(-\tfrac{1}{30})=\tfrac{1}{15}.$$

3. Let $u = 5x^2+4$ so that $du = 10x\,dx$ or $x\,dx = \frac{1}{10}\,du$. Also, if $x=0$, then $u=4$, and if $x=1$, then $u=9$. So

$$\int_0^1 x\sqrt{5x^2+4}\,dx = \tfrac{1}{10}\int_4^9 u^{1/2}\,du = \left.\tfrac{1}{15}u^{3/2}\right|_4^9 = \tfrac{1}{15}(27)-\tfrac{1}{15}(8)=\tfrac{19}{15}.$$

4. Let $u = 3x^2-2$ so that $du = 6x\,dx$ or $x\,dx = \frac{1}{6}\,du$. Also, if $x=1$, then $u=1$, and if $x=3$, then $u=25$. So,

$$\int_1^3 x\sqrt{3x^2-2}\,dx = \frac{1}{6}\int_1^{25} u^{1/2}du = \left.\frac{1}{9}u^{3/2}\right|_1^{25} = \frac{1}{9}(125)-\frac{1}{9}(1)=\frac{124}{9}.$$

5. Let $u = x^3 + 1$ so that $du = 3x^2\,dx$ or $x^2\,dx = \frac{1}{3}\,du$. Also, if $x = 0$, then $u = 1$, and if $x = 2$, then $u = 9$. So,

$$\int_0^2 x^2(x^3+1)^{3/2}\,dx = \frac{1}{3}\int_1^9 u^{3/2}\,du = \frac{2}{15}u^{5/2}\Big|_1^9 = \frac{2}{15}(243) - \frac{2}{15}(1) = \frac{484}{15}.$$

6. Let $u = 2x - 1$ so that $du = 2\,dx$ or $dx = \frac{1}{2}\,du$. Also, if $x = 1$, then $u = 1$ and if $x = 5$ then $u = 9$. So

$$\int_1^5 (2x-1)^{5/2}\,dx = \frac{1}{2}\int_1^9 u^{5/2}\,du = \frac{1}{7}u^{7/2}\Big|_1^9 = \frac{1}{7}(2187) - \frac{1}{7}(1) = \frac{2186}{7}.$$

7. Let $u = 2x + 1$ so that $du = 2\,dx$ or $dx = \frac{1}{2}\,du$. Also, if $x = 0$, then $u = 1$ and if $x = 1$ then $u = 3$. So

$$\int_0^1 \frac{1}{\sqrt{2x+1}}\,dx = \frac{1}{2}\int_1^3 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^3 u^{-1/2}\,du = u^{1/2}\Big|_1^3 = \sqrt{3} - 1.$$

8. Let $u = x^2 + 5$ so that $du = 2x\,dx$ or $x\,dx = \frac{1}{2}\,du$. Also, if $x = 0$ then $u = 5$ and if $x = 2$ then $u = 9$. So

$$\int_0^2 \frac{x}{\sqrt{x^2+5}}\,dx = \frac{1}{2}\int_5^9 \frac{du}{\sqrt{u}} = u^{1/2}\Big|_5^9 = 3 - \sqrt{5}.$$

9. $\int_1^2 (2x-1)^4\,dx$. Put $u = 2x - 1$ so that $du = 2\,dx$ or $dx = \frac{1}{2}\,du$.

Then $\int_1^2 (2x-1)^4\,dx = \frac{1}{2}\int_1^3 u^4\,du = \frac{1}{10}u^5\Big|_1^3 = \frac{1}{10}(243-1) = \frac{121}{5} = 24\frac{1}{5}$.

10. Let $u = x^2 + 4x - 8$ so that $du = (2x + 4)\,dx$. Also, if $x = 1$ then $u = -3$ and if $x = 2$, then $u = 4$. So

$$\int_1^2 (2x+4)(x^2+4x-8)^3\,dx = \int_{-3}^4 u^3\,du = \frac{1}{4}u^4\Big|_{-3}^4 = \frac{1}{4}(256) - \frac{1}{4}(81) = \frac{175}{4}.$$

11. Let $u = x^3 + 1$ so that $du = 3x^2\,dx$ or $x^2\,dx = \frac{1}{3}\,du$. Also, if $x = -1$, then $u = 0$ and if $x = 1$, then $u = 2$. So

$$\int_{-1}^1 x^2(x^3+1)^4\,dx = \frac{1}{3}\int_0^2 u^4\,du = \frac{1}{15}u^5\Big|_0^2 = \frac{32}{15}.$$

12. Let $u = x^4 + 3x$ so that $du = (4x^3 + 3)\,dx = 4(x^3 + \frac{3}{4})\,dx$ or $dx = (x^3 + \frac{3}{4}) = \frac{1}{4}\,du$. Also, if $x = 1$, then $u = 4$ and if $x = 2$, then $u = 22$. So

$$\int_1^2 \left(x^3 + \tfrac{3}{4}\right)(x^4 + 3x)^{-2}\,dx = \frac{1}{4}\int_4^{22} u^{-2}\,du = -\frac{1}{4u}\bigg|_4^{22} = -\tfrac{1}{88} + \tfrac{1}{16} = \tfrac{-2+11}{176} = \tfrac{9}{176}.$$

13. Let $u = x - 1$ so that $du = dx$. Then if $x = 1$, $u = 0$, and if $x = 5$, then $u = 4$.

$$\int_1^5 x\sqrt{x-1}\,dx = \int_0^4 (u+1)u^{1/2}\,du = \int_0^4 (u^{3/2} + u^{1/2})du$$

$$= \tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\Big|_0^4 = \tfrac{2}{5}(32) + \tfrac{2}{3}(8) = 18\tfrac{2}{15}.$$

14. Let $u = x + 1$ so that $du = dx$ and also $x = u - 1$. If $x = 1$, then $u = 2$ and if $x = 4$, then $u = 5$. So

$$\int_1^4 x\sqrt{x+1}\,dx = \int_2^5 (u-1)\sqrt{u}\,du = \int_2^5 (u^{3/2} - u^{1/2})du$$

$$= \tfrac{2}{5}u^{5/2} - \tfrac{2}{3}u^{3/2}\Big|_2^5 = \tfrac{2}{15}u^{3/2}(3u-5)\Big|_2^5 = \tfrac{2}{15}(50\sqrt{5} - 2\sqrt{2}).$$

15. Let $u = x^2$ so that $du = 2x\,dx$ or $x\,dx = \frac{1}{2}\,du$. If $x = 0$, $u = 0$ and if $x = 2$, $u = 4$. So

$$\int_0^2 xe^{x^2}\,dx = \tfrac{1}{2}\int_0^4 e^u\,du = \tfrac{1}{2}e^u\Big|_0^4 = \tfrac{1}{2}(e^4 - 1).$$

16. Let $u = -x$ so that $du = -dx$ or $dx = -du$. If $x = 0$, $u = 0$ and if $x = 1$, $u = -1$. So

$$\int_0^1 e^{-x}\,dx = -\int_0^{-1} e^u\,du = -e^u\Big|_0^{-1} = -e^{-1} + 1 = 1 - \frac{1}{e}.$$

17. $$\int_0^1 (e^{2x} + x^2 + 1)\,dx = \tfrac{1}{2}e^{2x} + \tfrac{1}{3}x^3 + x\Big|_0^1 = (\tfrac{1}{2}e^2 + \tfrac{1}{3} + 1) - \tfrac{1}{2}$$

$$= \frac{1}{2}e^2 + \frac{5}{6}.$$

18. $$\int_0^2 (e^t - e^{-t})\,dt = e^t + e^{-t}\Big|_0^2 = (e^2 + e^{-2}) - (1+1) = e^2 + e^{-2} - 2.$$

19. Put $u = x^2 + 1$ so that $du = 2x\,dx$ or $x\,dx = \frac{1}{2}\,du$. Then

$$\int_{-1}^{1} xe^{x^2+1}dx = \frac{1}{2}\int_{2}^{2} e^u\, du = \frac{1}{2}e^u\Big|_2^2 = 0$$

(Since the upper and lower limits are equal.)

20. Let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}dx$. If $x = 0$, $u = 0$, and if $x = 4$, $u = 2$.

$$\int_0^4 \frac{e\sqrt{x}}{\sqrt{x}}dx = 2\int_0^2 e^u\, du = 2e^u\Big|_0^2 = 2(e^2 - 1).$$

21. Let $u = x - 2$ so that $du = dx$. If $x = 3$, $u = 1$ and if $x = 6$, $u = 4$. So

$$\int_3^6 \frac{2}{x-2}dx = 2\int_1^4 \frac{du}{u} = 2\ln|u|\Big|_1^4 = 2\ln 4.$$

22. Let $u = 1 + 2x^2$ so that $du = 4x\, dx$ or $x\, dx = \frac{1}{4}du$. If $x = 0$, $u = 1$ and if $x = 1$, $u = 3$. So

$$\int_0^1 \frac{x}{1+2x^2}dx = \frac{1}{4}\int_1^3 \frac{du}{u} = \frac{1}{4}\ln|u|\Big|_1^3 = \frac{1}{4}\ln 3.$$

23. Let $u = x^3 + 3x^2 - 1$ so that $du = (3x^2 + 6x)dx = 3(x^2 + 2x)dx$. If $x = 1$, $u = 3$, and if $x = 2$, $u = 19$. So

$$\int_1^2 \frac{x^2+2x}{x^3+3x^2-1}dx = \frac{1}{3}\int_3^{19} \frac{du}{u} = \frac{1}{3}\ln u\Big|_3^{19} = \frac{1}{3}(\ln 19 - \ln 3).$$

24. $$\int_0^1 \frac{e^x}{1+e^x}dx = \ln(1+e^x)\Big|_0^1 = \ln(1+e) - \ln 2 = \ln\left(\frac{1+e}{2}\right).$$

25. $$\int_1^2 \left(4e^{2u} - \frac{1}{u}\right)du = 2e^{2u} - \ln u\Big|_1^2 = (2e^4 - \ln 2) - (2e^2 - 0) = 2e^4 - 2e^2 - \ln 2.$$

26. $$\int_1^2 \left(1 + \frac{1}{x} + e^x\right)dx = x + \ln x + e^x\Big|_1^2 = (2 + \ln 2 + e^2) - (1 + e)$$

$$= 1 + \ln 2 + e^2 - e.$$

27. $\int_1^2 (2e^{-4x} - x^{-2})dx = -\frac{1}{2}e^{-4x} + \frac{1}{x}\Big|_1^2 = (-\frac{1}{2}e^{-8} + \frac{1}{2}) - (-\frac{1}{2}e^{-4} + 1)$

$$= -\frac{1}{2}e^{-8} + \frac{1}{2}e^{-4} - \frac{1}{2} = \frac{1}{2}(e^{-4} - e^{-8} - 1).$$

28. Let $u = \ln x$, $du = \frac{1}{x}dx$. If $x = 1$, $u = 0$ and if $x = 2$, $u = \ln 2$.

So $\int_1^2 \frac{\ln x}{x} dx = \int_0^{\ln 2} u\, du = \frac{1}{2}u^2\Big|_0^{\ln 2} = \frac{1}{2}(\ln 2)^2.$

29. AV $= \frac{1}{2}\int_0^2 (2x+3)\, dx = \frac{1}{2}(x^2 + 3x)\Big|_0^2 = \frac{1}{2}(10) = 5.$

30. AV $= \frac{1}{b-a}\int_a^b f(x)\, dx = \frac{1}{4-1}\int_1^4 (8-x)\, dx = \frac{1}{3}\int_1^4 (8-x)\, dx$

$= \frac{1}{3}(8x - \frac{1}{2}x^2)\Big|_1^4 = \frac{1}{3}[(32-8) - (8-\frac{1}{2})] = 5\frac{1}{2}.$

31. AV $= \frac{1}{2}\int_1^3 (2x^2 - 3)\, dx = \frac{1}{2}(\frac{2}{3}x^3 - 3x)\Big|_1^3 = \frac{1}{2}(9 + \frac{7}{3}) = \frac{17}{3}.$

32. AV $= \frac{1}{5}\int_{-2}^3 (4 - x^2)\, dx = \frac{1}{5}(4x - \frac{1}{3}x^3)\Big|_{-2}^3 = \frac{1}{5}[(12-9) - (-8 + \frac{8}{3})] = \frac{5}{3}.$

33. AV $= \frac{1}{3}\int_{-1}^2 (x^2 + 2x - 3)\, dx = \frac{1}{3}(\frac{1}{3}x^3 + x^2 - 3x)\Big|_{-1}^2$

$= \frac{1}{3}[(\frac{8}{3} + 4 - 6) - (-\frac{1}{3} + 1 + 3)] = \frac{1}{3}(\frac{8}{3} - 2 + \frac{1}{3} - 4) = -1.$

34. AV $= \frac{1}{2}\int_{-1}^1 x^3\, dx = \frac{1}{2}(\frac{1}{4}x^4)\Big|_{-1}^1 = \frac{1}{2}(\frac{1}{4} - \frac{1}{4}) = 0$

35. AV $= \frac{1}{4}\int_0^4 (2x+1)^{1/2}\, dx = (\frac{1}{4})(\frac{1}{2})(\frac{2}{3})(2x+1)^{3/2}\Big|_0^4 = \frac{1}{12}(27-1) = \frac{13}{6}$

36. $\text{AV} = \frac{1}{4-0}\int_0^4 e^{-x}\,dx = -\frac{1}{4}e^{-x}\Big|_0^4 = -\frac{1}{4}(e^{-4}-1) \approx 0.245.$

37. $\text{AV} = \frac{1}{2}\int_0^2 xe^{x^2}\,dx = \frac{1}{4}e^{x^2}\Big|_0^2 = \frac{1}{4}(e^4-1).$

38. $\text{AV} = \frac{1}{2}\int_0^2 \frac{dx}{x+1} = \frac{1}{2}\ln(x+1)\Big|_0^2 = \frac{1}{2}\ln 3.$

39. The amount produced was

$$\int_0^{20} 3.5e^{0.05t}\,dt = \frac{3.5}{0.05}e^u\Big|_0^{20} \qquad \text{(Use the substitution } u = 0.05t.\text{)}$$

$$= 70(e-1) \approx 120.3, \quad \text{or 120.3 billion metric tons.}$$

40. The temperature will have dropped

$$\int_0^3 -18e^{-0.6t}\,dt = \frac{-18}{-0.6}e^{-0.6t}\Big|_0^3 \qquad \text{(Use the substitution } u = 0.05t.\text{)}$$

$$= 30e^{-0.6t}\Big|_0^3 = 30(e^{-1.8}-1) = 25.04, \text{ or approximately 25 degrees.}$$

$f(t) = 30e^{-0.6t} + C$; $f(0) = 30 + C = 68$, and $C = 38$.

The temperature of the wine at 7 P.M. is

$f(3) = 30e^{-1.8} + 38 \approx 42.96$, or approximately 43° F.

41. The amount is $\int_1^2 t(\frac{1}{2}t^2+1)^{1/2}\,dt$. Let $u = \frac{1}{2}t^2+1$, so that $du = t\,dt$. Therefore,

$$\int_1^2 t(\tfrac{1}{2}t^2+1)^{1/2}\,dt = \int_{3/2}^3 u^{1/2}\,du = \tfrac{2}{3}u^{3/2}\Big|_{3/2}^3 = \tfrac{2}{3}[(3)^{3/2} - (\tfrac{3}{2})^{3/2}]$$

$$\approx 2.24 \text{ million dollars.}$$

42. The amount of oil that the well can be expected to yield is

$$\int_0^5\left(\frac{600t^2}{t^3+32}+5\right)dt = 600\int_0^5 \frac{t^2}{t^3+32}\,dt + 5t\Big|_0^5 = 600\left(\frac{1}{3}\right)\ln(t^3+32)\Big|_0^5 + 25$$

$$= 200(\ln 157 - \ln 32) + 25 \approx 343$$

or 343 thousand barrels.

43. The tractor will depreciate

$$\int_0^5 13388.61e^{-0.22314t}\,dt = \frac{13388.61}{-0.22314}e^{-0.22314t}\Big|_0^5$$

$$= -60{,}000.94e^{-0.22314t}\Big|_0^5 = -60{,}000.94(-0.672314)$$

$$= 40{,}339.47, \quad \text{or } \$40{,}339.$$

44. The distance traveled is $\int_0^4 3t\sqrt{16-t^2}\,dt = 3(-\tfrac{1}{2})(\tfrac{2}{3})(16-t^2)^{3/2}\Big|_0^4 = 64$ ft.

45. $$\overline{A} = \tfrac{1}{5}\int (\tfrac{1}{12}t^2 + 2t + 44)dt = \tfrac{1}{5}\left[\tfrac{1}{36}t^3 + t^2 + 44t\Big|_0^5\right]$$

$$= \tfrac{1}{5}\left[\tfrac{125}{36} + 25 + 220\right] = \frac{125+900+7920}{5(36)} \approx 49.69, \text{ or } 49.7 \text{ ft/sec.}$$