MATH180 – HOMEWORK SOLUTIONS

HOMEWORK #7

Section 5.6: 1, 5, 9, 15, 21, 30

Section 6.1: 1-61 (odd only), 69, 73

Section 6.2: 1-53 (odd only), 59, 61

EXERCISES 5.6 , page 388

1. a. The growth constant is $k = 0.05$. b. Initially, the quantity present is 400 units.
c.

t	0	10	20	100	1000
Q	400	660	1087	59365	2.07×10^{24}

5. a. We solve the equation

$$5.3e^{0.0198t} = 3(5.3) \text{ or } e^{0.0198t} = 3,$$

or $$0.0198t = \ln 3 \text{ and } t = \frac{\ln 3}{0.0198} \approx 55.5.$$

So the world population will triple in approximately 55.5 years.

b. If the growth rate is 1.8 percent, then proceeding as before, we find

$1.018(5.3) = 5.3e^{k}$, and $k = \ln 1.018 \approx 0.0178$.

So $N(t) = 5.3e^{0.0178t}$. If $t = 55.5$, the population would be

$$N(55.5) = 5.3e^{0.0178(55.5)} \approx 14.23, \text{ or approximately 14.23 billion.}$$

6. The resale value of the machinery at any time t is given by $V(t) = 500{,}000e^{-kt}$ ($t = 0$ three years ago). We have $V(3) = 320{,}000 = 500{,}000e^{-3k}$ which gives $e^{-3k} = \frac{320{,}000}{500{,}000} = 0.64$. Therefore, $-3k \ln e = \ln 0.64$ and $k = \frac{\ln 0.64}{-3} \approx 0.149$. Four years from now, the resale value of the machinery will be given by

$$V(7) = 500{,}000e^{-(0.149)(7)} \approx 176{,}198, \text{ or approximately \$176,198.}$$

7. $P(h) = p_0 e^{-kh}$, $P(0) = 15$, therefore, $p_0 = 15$.

$$P(4000) = 15e^{-4000k} = 12.5; \quad e^{-4000k} = \frac{12.5}{15},$$

$$-4000k = \ln\left(\frac{12.5}{15}\right) \text{ and } k = 0.00004558.$$

Therefore, $P(12{,}000) = 15e^{-0.00004558(12{,}000)} = 8.68$, or 8.7 lb/sq in.

The rate of change of the atmospheric pressure with respect to altitude is given by

$$P'(h) = \frac{d}{dh}(15e^{-0.00004558h}) = -0.0006837e^{-0.00004558h}.$$

So, the rate of change of the atmospheric pressure with respect to altitude when the altitude is 12,000 feet is $P'(12{,}000) = -0.0006837e^{-0.00004558(12{,}000)} \approx -0.00039566$.

That is, it is dropping at the rate of approximately 0.0004 lbs per square inch/foot.

8. We are given that Q(280) = 20. Using this condition, we have

$Q(280) = Q_0 \bullet 2^{(-280/140)} = 20$. So $Q_0 \bullet 2^{-2} = 20$ or $\frac{Q_0}{4} = \frac{20}{4} = 5$. So the initial amount was 80 mg.

9. Suppose the amount of phosphorus 32 at time t is given by

$$Q(t) = Q_0 e^{-kt}$$

where Q_0 is the amount present initially and k is the decay constant. Since this element has a half–life of 14.2 days, we have

$$\tfrac{1}{2}Q_0 = Q_0 e^{-14.2k}, \quad e^{-14.2k} = \tfrac{1}{2}, \; -14.2k = \ln\tfrac{1}{2}, \; k = -\frac{\ln\frac{1}{2}}{14.2} \approx 0.0488.$$

Therefore, the amount of phosphorus 32 present at any time t is given by

$$Q(t) = 100e^{-0.0488t}$$

The amount left after 7.1 days is given by

$$Q(7.1) = 100e^{-0.0488(7.1)} = 100e^{-0.3465}$$
$$= 70.617, \quad \text{or } 70.617 \text{ grams.}$$

The rate at which the phosphorus 32 is decaying when $t = 7.1$ is given by

$$Q'(t) = \frac{d}{dt}[100e^{-0.0488t}] = 100(-0.0488)e^{-0.0488t} = -4.88e^{-0.0488t}.$$

Therefore, $Q'(7.1) = -4.88e^{-0.0488(7.1)} \approx -3.451$; that is, it is changing at the rate of 3.451 gms/day.

10. Suppose the amount of strontium 90 present at time t is given by $Q(t) = Q_0 e^{-kt}$ where Q_0 is the amount present initially and k is the decay constant. Since this element has a half-life of 27 years, we find

$$\tfrac{1}{2}Q_0 = Q_0 e^{-27k}, \quad e^{-27k} = \tfrac{1}{2}, \; -27k = \ln\tfrac{1}{2}, \; k = -\tfrac{1}{27}\ln\tfrac{1}{2}.$$

Therefore, the amount of strontium 90 present at any time t is given by

$$Q(t) = Q_0 e^{((1/27)\ln 1/2)t} = Q_0 e^{\ln(1/2)(t/27)} = Q_0(1/2)^{t/27}.$$

We want to find t when $Q(t) = (1/4)Q_0$.

$$\frac{1}{4}Q_0 = Q_0\left(\frac{1}{2}\right)^{t/27}, \quad \left(\frac{1}{2}\right)^{t/27} = \frac{1}{4}, \quad \frac{t}{27}\ln\frac{1}{2} = \ln\frac{1}{4},$$

$$t = 27\frac{\ln\frac{1}{4}}{\ln\frac{1}{2}} = 27\left(\frac{-\ln 4}{-\ln 2}\right) \approx 54, \text{ or approximately 54 years.}$$

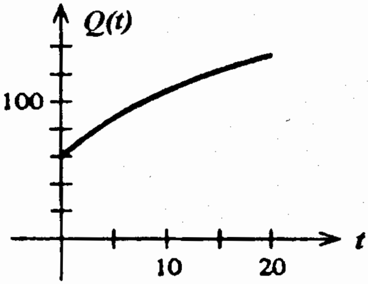
11. We solve the equation $0.2Q_0 = Q_0 e^{-0.00012t}$

obtaining $$t = \frac{\ln 0.2}{-0.00012} \approx 13{,}412, \text{ or approximately 13,412 years.}$$

12. We solve the equation $0.18Q_0 = Q_0 e^{-0.00012t}$ giving

$$t = \frac{\ln 0.18}{-0.00012} \approx 14{,}290, \quad \text{or approximately 14,290 years.}$$

13. The graph of $Q(t)$ follows.



a. $Q(0) = 120(1 - e^0) + 60 = 60$, or 60 w.p.m.
b. $Q(10) = 120(1 - e^{-0.5}) + 60 = 107.22$, or approximately 107 w.p.m.
c. $Q(20) = 120(1 - e^{-1}) + 60 = 135.65$, or approximately 136 w.p.m.

14. a. $S(t) = 50{,}000 + Ae^{-kt}$. Using the condition $S(1) = 83{,}515$ and $S(3) = 65{,}055$, we have $S(1) = 50{,}000 + Ae^{-k} = 83{,}515$ and $S(3) = 50{,}000 + Ae^{-3k} = 65{,}055$. The first equation gives $Ae^{-k} = 33{,}515$ and the second equation gives $Ae^{-3k} = 15055$. So

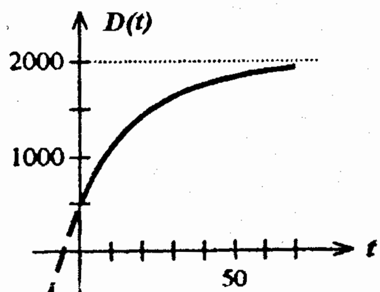
$$\frac{Ae^{-k}}{Ae^{-3k}} = \frac{33{,}515}{15{,}055},\quad e^{2k} = \frac{33{,}515}{15{,}055},\ \text{or}\ k = \frac{1}{2}\ln\left(\frac{33{,}515}{15{,}055}\right) \approx 0.40014.$$

b. $A = 33515e^{-k} = 33515e^{-0.40014} = 22463$. So $S(t) = 50{,}000 + 224633e^{-0.40014t}$.
In particular, $S(4) = 50{,}000 + 22463e^{-0.40014(4)} \approx 54{,}533$,
or approximately 54,533.

c. $$S'(t) = \frac{d}{dt}[50{,}000 + 22463e^{-0.40014t}]$$
$$= 22{,}463(-0.40014)e^{-0.40014t} = -8988.34e^{-0.40014t}$$

and so $S'(t) = -8988.34e^{-0.40014(4)} \approx -1813.7$.
That is, the sales volume is dropping at the rate of approximately \$1814/week.

15. The graph of $D(t)$ follows.



a. After one month, the demand is $D(1) = 2000 - 1500e^{-0.05} \approx 573$.
After twelve months, the demand is $D(12) = 2000 - 1500e^{-0.6} \approx 1177$.
After twenty-four months the demand is $D(24) = 2000 - 1500e^{-1.2} \approx 1548$.
After sixty months, the demand is $D(60) = 2000 - 1500e^{-3} \approx 1925$.

b. $$\lim_{t\to\infty} D(t) = \lim_{t\to\infty} 2000 - 1500e^{-0.05t} = 2000$$

and we conclude that the demand is expected to stabilize at 2000 computers per month.

c. $D'(t) = -1500e^{-0.05t}(-0.05) = 75e^{-0.05t}$. Therefore, the rate of growth after ten months is given by $D'(10) = 75e^{-0.5} \approx 45.49$, or approximately 46 computers per month.

16. a. The percent that will fail after 3 years is $P(3) = 100(1 - e^{-0.3}) \approx 25.92$. Therefore, 74% will be usable.

b. $\lim_{t\to\infty} P(t) = \lim_{t\to\infty} 100(1 - e^{-0.1t}) = 100$. So all will fail eventually, as one might expect.

17. a. The length is given by $f(5) = 200(1 - 0.956e^{-0.18(5)}) \approx 122.26$, or approximately 122.3 cm.

b. $f'(t) = 200(-0.956)e^{-0.18t}(-0.18) = 34.416e^{-0.18t}$. So, a 5-yr old is growing at the rate of $f'(5) = 34.416e^{-0.18(5)} \approx 13.9925$, or approximately 14 cm/yr.

c. The maximum length is given by $\lim_{t\to\infty} 200(1 - 0.956e^{-0.18t}) = 200$, or 200 cm.

18. a. $Q(1) = \dfrac{1000}{1 + 199e^{-0.8}} \approx 11.06$, or 11 children.

b. $Q(10) = \dfrac{1000}{1 + 199e^{-8}} \approx 937.4$, or 937 children.

c. $\lim_{t\to\infty} \dfrac{1000}{1 + 199e^{-0.8t}} = 1000$, or 1000 children.

19. a. The percent of lay teachers is $f(3) = \dfrac{98}{1 + 2.77e^{-3}} \approx 86.1228$, or 86.12%.

b. $$f'(t) = \frac{d}{dt}[98(1 + 2.77e^{-t})^{-1}] = 98(-1)(1 + 2.77e^{-t})^{-2}(2.77e^{-t})(-1)$$
$$= \frac{271.46e^{-t}}{(1 + 2.77e^{-t})^2}$$

$$f'(3) = \frac{271.46e^{-3}}{(1+2.77e^{-3})^2} \approx 10.4377.$$

So it is increasing at the rate of 10.44%/yr.

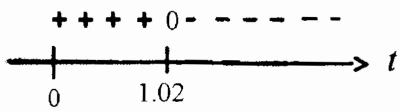
c. $$f''(t) = 271.46\left[\frac{(1+2.77e^{-t})^2(-e^{-t}) - e^{-t}\cdot 2(1+2.77e^{-t})(-2.77e^{-t})}{(1+2.77e^{-t})^4}\right]$$

$$= \frac{271.46[-(1+2.77e^{-t}+5.54e^{-t}]}{e^{t}(1+2.77e^{-t})^3} = \frac{271.46(2.77e^{-t}-1)}{e^{t}(1+2.77e^{-t})^3}.$$

Setting $f''(t) = 0$ gives $2.77e^{-t} = 1$

$$e^{-t} = \frac{1}{2.77}; \quad -t = \ln\left(\tfrac{1}{2.77}\right), \quad \text{and} \quad t = 1.0188.$$

The sign diagram of f'' shows that $t = 1.02$ gives an inflection point of P. So, the



percent of lay teachers was increasing most rapidly in 1970.

20. a. $N(0) = \dfrac{400}{1+39} = 10$ flies. b. $\lim\limits_{t\to\infty} \dfrac{400}{1+39e^{-0.16t}} = 400$ flies.

c. $N(20) = \dfrac{400}{1+39e^{-0.16(20)}} \approx 154.5$, or 154 flies.

d. $$N'(t) = \frac{d}{dt}\left[400(1+39e^{-0.16t})^{-1}\right]$$

$$= -400(1+39e^{-0.16t})^{-2}\frac{d}{dt}(39e^{-0.16t})$$

$$= \frac{2496e^{-0.16t}}{(1+39e^{-0.16t})^2}$$

$N'(20) = \dfrac{2496e^{-0.16(20)}}{(1+39e^{-0.16(20)})^2} \approx 15.17031574$, or approximately 15 fruit-flies per day.

21. $P(t) = \dfrac{68}{1+21.67e^{-0.62t}}$.

The percentage of households that owned VCRs at the beginning of 1985 is given by $P(0) = \dfrac{68}{1+21.67e^{-0.62(0)}} = \dfrac{68}{22.67} \approx 3$, or approximately 3 percent.

The percentage of households that owned VCRs at the beginning of 1995 is given by $P(10) = \dfrac{68}{1+21.67e^{-0.62(10)}} \approx 65.14$, or approximately 65.14 percent.

22. The expected population of the U. S. in 2020 is $P(3) = \dfrac{616.5}{1+4.02e^{-0.5(3)}} \approx 324.99$, or approximately 325 million people.

23. The first of the given conditions implies that $f(0) = 300$, that is,

$$300 = \frac{3000}{1+Be^0} = \frac{3000}{1+B}.$$

So $1+B = 10$, or $B = 9$. Therefore, $f(t) = \dfrac{3000}{1+9e^{-kt}}$. Next, the condition $f(2) = 600$ gives the equation

$$600 = \frac{3000}{1+9e^{-2k}},\quad 1+9e^{-2k} = 5,\ e^{-2k} = \frac{4}{9},\quad \text{or } k = -\frac{1}{2}\ln\left(\frac{4}{9}\right).$$

Therefore, $f(t) = \dfrac{3000}{1+9e^{(1/2)t\cdot\ln(4/9)}} = \dfrac{3000}{1+9(\frac{4}{9})^{t/2}}$.

The number of students who had heard about the policy four hours later is given by

$$f(4) = \frac{3000}{1+9(\frac{4}{9})^2} = 1080, \quad \text{or 1080 students.}$$

To find the rate at which the rumor was spreading at any time time, we compute

$$f'(t) = \frac{d}{dt}\left[3000(1+9e^{-0.405465t})^{-1}\right]$$

$$= (3000)(-1)(1+9e^{-0.405465})^{-2}\frac{d}{dt}(9e^{-0.405465t})$$

$$= -3000(9)(-0.405465)e^{-0.405465t}(1+9e^{-0.405465t})^{-2}$$

$$= \frac{10947.555\,e^{-0.405465t}}{(1+9e^{-0.405465t})^2}$$

In particular, the rate at which the rumor was spreading 4 hours after the ceremony is given by $f'(4) = \dfrac{10947.555e^{-0.405465(4)}}{(1+9e^{-0.405465(4)})^2} \approx 280.25737$.

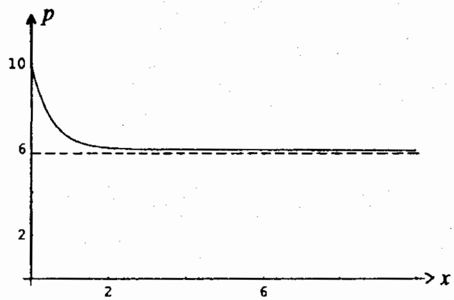
So , the rumor is spreading at the rate of 280 students per hour.

24. a. $f'(t) = -8e^{-2t} < 0$ for all t in $(0,\infty)$. So f is decreasing on $(0,\infty)$.

b. $f''(t) = 16e^{-2t} > 0$ for all t in $(0,\infty)$. So, f is concave upward on $(0,\infty)$.

c. $\lim_{t\to\infty} f(t) = \lim_{t\to\infty}(6+4e^{-2t}) = 6.$

d.



25. $x(t) = \dfrac{15\left(1-(\frac{2}{3})^{3t}\right)}{1-\frac{1}{4}(\frac{2}{3})^{3t}}$; $\lim_{t\to\infty} x(t) = \lim_{t\to\infty}\dfrac{15\left(1-(\frac{2}{3})^{3t}\right)}{1-\frac{1}{4}(\frac{2}{3})^{3t}} = \dfrac{15(1-0)}{1-0} = 15$

or 15 lbs.

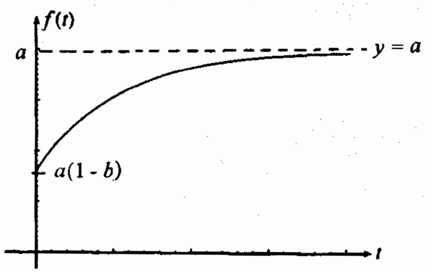
26. a. $f'(t) = \dfrac{d}{dt}[a(1-be^{-kt})] = \dfrac{d}{dt}(a) - \dfrac{d}{dt}abe^{-kt} = 0 - be^{-kt}(-k) = bke^{-kt}.$

Since $f'(t) > 0$ for all $t \geq 0$, f is increasing on $(0,\infty)$.

b. $f''(t) = \dfrac{d}{dt}(bke^{-kt}) = -bk^2e^{-kt} < 0$ on $(0,\infty)$ and the conclusion follows.

c. $\lim_{t\to\infty} f(t) = \lim_{t\to\infty}[a(1-be^{-kt})] = \lim_{t\to\infty} a - \lim_{t\to\infty} abe^{-kt} = a - 0 = a.$

d.



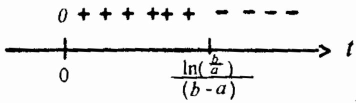
27. a. $C(t) = \frac{k}{b-a}\left(e^{-at} - e^{-bt}\right)$;

$$C'(t) = \frac{k}{b-a}(-ae^{-at} + be^{-bt}) = \frac{kb}{b-a}\left[e^{-bt} - \left(\frac{a}{b}\right)e^{-at}\right]$$

$$= \frac{kb}{b-a}e^{-bt}\left[1 - \frac{a}{b}e^{(b-a)t}\right]$$

$C'(t) = 0$ implies that $1 = \frac{a}{b}e^{(b-a)t}$ or $t = \frac{\ln\left(\frac{b}{a}\right)}{b-a}$.

The sign diagram of C'



shows that this value of t gives a minimum.

b. $\lim_{t\to\infty} C(t) = \frac{k}{b-a}$.

28. a. $\lim_{t\to\infty}\left\{\frac{r}{k} - \left[\left(\frac{r}{k}\right) - C_0\right]e^{-kt}\right\} = \frac{r}{k}$, and this shows that in the long run the concentration of the glucose solution approaches r/k.

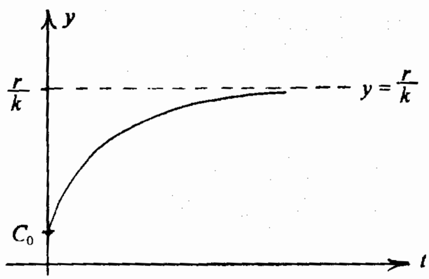
b. $C'(t) = -\left[\left(\frac{r}{k}\right) - C_0\right]e^{-kt}(-k) = k\left[\left(\frac{r}{k}\right) - C_0\right]e^{-kt} > 0$ (since $\frac{r}{k} > C_0$)

for all $t > 0$. So, C is increasing on $(0, \infty)$.

c. $C''(t) = -k^2\left[\left(\frac{r}{k}\right) - C_o\right]e^{-kt} < 0$ (sin *ce* $\frac{r}{k} > C_0$)

for all $t > 0$. So, the graph of C is concave upward.

d.



29. a. We solve $Q_0e^{-kt} = \frac{1}{2}Q_0$ for t. Proceeding, we have

$$e^{-kt} = \frac{1}{2},\ \ln e^{-kt} = \ln\frac{1}{2} = \ln 1 = \ln 2 = -\ln 2;$$

$$-kt = -\ln 2;$$

So $$\bar{t} = \frac{\ln 2}{k}$$

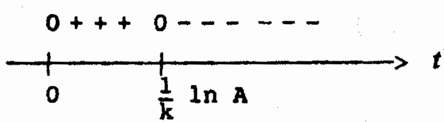
b. $$\bar{t} = \frac{\ln 2}{0.0001238} \approx 5598.927,\ \text{or approximately 5599 years.}$$

30. a. $Q'(t) = Ce^{-Ae^{-kt}} \dfrac{d}{dt}(-Ae^{-kt}) = -ACe^{-Ae^{-kt}} \cdot e^{-kt}(-k) = ACke^{(-Ae^{-kt}-kt)}$.

b. $Q''(t) = ACke^{(-Ae^{-kt}-kt)} \cdot [-k - Ae^{-kt}(-k)] = 0$, if $Ae^{-kt} = 1$.

$$e^{-kt} = \frac{1}{A},\quad -kt = \ln\frac{1}{A},\quad \text{or } t = -\frac{1}{k}\ln\frac{1}{A} = \frac{1}{k}\ln \text{A}.$$

The sign diagram shows that $t = \frac{1}{k}\ln A$ is an inflection point and so the growth is most rapid at this time.



c. $\lim_{t\to\infty} Q(t) = C$.

CHAPTER 6

EXERCISES 6.1, page 407

1. $F(x) = \frac{1}{3}x^3 + 2x^2 - x + 2$; $F'(x) = x^2 + 4x - 1 = f(x)$.

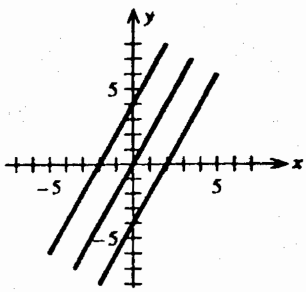
2. $F(x) = xe^x + \pi$; $F'(x) = xe^x + e^x = e^x(x+1) = f(x)$.

3. $F(x) = (2x^2 - 1)^{1/2}$; $F'(x) = \frac{1}{2}(2x^2 - 1)^{-1/2}(4x) = 2x(2x^2 - 1)^{-1/2} = f(x)$.

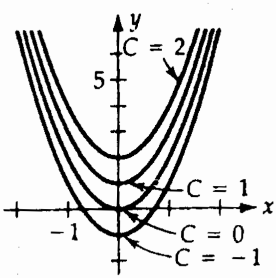
4. $F(x) = x\ln x - x$; $F'(x) = x(\frac{1}{x}) + \ln x - 1 = \ln x = f(x)$.

5. a. $G'(x) = \dfrac{d}{dx}(2x) = 2 = f(x)$ b. $F(x) = G(x) + C = 2x + C$

   c.

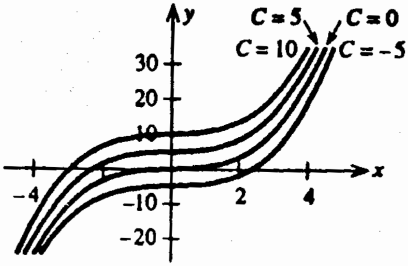


6. a. $G'(x) = 4x = f(x)$ and so G is an antiderivative of f.
   b. $H(x) = G(x) + C = 2x^2 + C$, where C is an arbitrary constant.
   c.

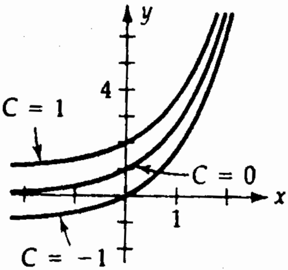


7. a. $G'(x) = \dfrac{d}{dx}(\frac{1}{3}x^3) = x^2 = f(x)$ b. $F(x) = G(x) + C = \frac{1}{3}x^3 + C$

c.



8. a. $G(x) = e^x$ and $G'(x) = e^x = f(x)$. b. $F(x) = e^x + C$, C an arbitrary constant.
c.



9. $\int 6\, dx = 6x + C.$

10. $\int \sqrt{2}\, dx = \sqrt{2}x + C$

11. $\int x^3 dx = \frac{1}{4}x^4 + C$

12. $\int 2x^5 dx = 2(\frac{1}{6}x^6) + C = \frac{1}{3}x^6 + C.$

13. $\int x^{-4} dx = -\frac{1}{3}x^{-3} + C$

14. $\int 3t^{-7} dt = 3(-\frac{1}{6}t^{-6}) + C = -\frac{1}{2}t^{-6} + C$

15. $\int x^{2/3} dx = \frac{3}{5}x^{5/3} + C$

16. $\int 2u^{3/4} du = 2(\frac{4}{7}u^{7/4}) + C = \frac{8}{7}u^{7/4} + C$

17. $\int x^{-5/4} dx = -4x^{-1/4} + C$

18. $\int 3x^{-2/3} dx = 3\left(\frac{x^{1/3}}{\frac{1}{3}}\right) + C = 9x^{1/3} + C$

19. $\int \frac{2}{x^2}\, dx = 2\int x^{-2} dx = 2(-1x^{-1}) + C = -\frac{2}{x} + C$

20. $\int \frac{1}{3x^5}\, dx = \frac{1}{3}\int x^{-5} dx = \frac{1}{3}(-\frac{1}{4}x^{-4}) + C = -\frac{1}{12x^4} + C$

21. $\int \pi\sqrt{t}\,dt = \pi\int t^{1/2}dt = \pi(\frac{2}{3}t^{3/2}) + C = \dfrac{2\pi}{3}t^{3/2} + C$

22. $\int \dfrac{3}{\sqrt{t}}\,dt = 3\int t^{-1/2}dt = 6t^{1/2} + C = 6\sqrt{t} + C$

23. $\int (3-2x)\,dx = \int 3\,dx - 2\int x\,dx = 3x - x^2 + C$

24. $\int (1+u+u^2)\,du = u + \frac{1}{2}u^2 + \frac{1}{3}u^3 + C$

25. $\int (x^2+x+x^{-3})\,dx = \int x^2\,dx + \int x\,dx + \int x^{-3}\,dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{2}x^{-2} + C$

26. $\int (0.3t^2 + 0.02t + 2)\,dt = 0.3(\frac{1}{3}t^3) + 0.02(\frac{1}{2}t^2) + 2t + C = 0.1t^3 + 0.01t^2 + 2t + C$

27. $\int 4e^x\,dx = 4e^x + C$

28. $\int (1+e^x)\,dx = x + e^x + C$

29. $\int (1+x+e^x)\,dx = x + \frac{1}{2}x^2 + e^x + C$

30. $\int (2+x+2x^2+e^x)\,dx = 2x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + e^x + C$

31. $\int (4x^3 - \dfrac{2}{x^2} - 1)\,dx = \int (4x^3 - 2x^{-2} - 1)\,dx = x^4 + 2x^{-1} - x + C = x^4 + \dfrac{2}{x} - x + C$

32. $\int (6x^3 + 3x^{-2} - x)\,dx = \dfrac{3}{2}x^4 - 3x^{-1} - \dfrac{1}{2}x^2 + C$

33. $\int (x^{5/2} + 2x^{3/2} - x)\,dx = \frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} - \frac{1}{2}x^2 + C$

34. $\int (t^{3/2} + 2t^{1/2} - 4t^{-1/2})\,dt = \frac{2}{5}t^{5/2} + \frac{4}{3}t^{3/2} - 8t^{1/2} + C$

35. $\int (x^{1/2} + 3x^{-1/2})\,dx = \frac{2}{3}x^{3/2} + 6x^{1/2} + C$

36. $\int (x^{2/3} - x^{-2})\,dx = \dfrac{3}{5}x^{5/3} + \dfrac{1}{x} + C$

37. $\int \left(\dfrac{u^3 + 2u^2 - u}{3u}\right)du = \dfrac{1}{3}\int (u^2 + 2u - 1)\,du = \dfrac{1}{9}u^3 + \dfrac{1}{3}u^2 - \dfrac{1}{3}u + C$

38. $\int (x^2 - x^{-2})\,dx = \dfrac{1}{3}x^3 + x^{-1} = \dfrac{1}{3}x^3 + \dfrac{1}{x} + C$

39. $\int (2t+1)(t-2)\,dt = \int (2t^2 - 3t - 2)\,dt = \frac{2}{3}t^3 - \frac{3}{2}t^2 - 2t + C$

40. $\int u^{-2}(1 - u^2 + u^4)\,du = \int (u^{-2} - 1 + u^2)\,du = -u^{-1} - u + \frac{1}{3}u^3 + C$

41. $\int \frac{1}{x^2}(x^4 - 2x^2 + 1)\,dx = \int (x^2 - 2 + x^{-2})\,dx = \frac{1}{3}x^3 - 2x - x^{-1} + C$

$$= \frac{1}{3}x^3 - 2x - \frac{1}{x} + C$$

42. $\int t^{1/2}(t^2 + t - 1)\,dt = \int (t^{5/2} + t^{3/2} - t^{1/2})\,dt = \frac{2}{7}t^{7/2} + \frac{2}{5}t^{5/2} - \frac{2}{3}t^{3/2} + C$

43. $\int \frac{ds}{(s+1)^{-2}} = \int (s+1)^2\,ds = \int (s^2 + 2s + 1)\,ds = \frac{1}{3}s^3 + s^2 + s + C$

44. $\int (x^{1/2} + 3x^{-1} - 2e^x)\,dx = \frac{2}{3}x^{3/2} + 3\ln|x| - 2e^x + C$

45. $\int (e^t + t^e)\,dt = e^t + \frac{1}{e+1}t^{e+1} + C$

46. $\int \left(\frac{1}{x^2} - \frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt{x}}\right)dx = \int (x^{-2} - x^{-2/3} + x^{-1/2})\,dx$

$$= -x^{-1} - 3x^{1/3} + 2x^{1/2} + C = -\frac{1}{x} - 3x^{1/3} + 2\sqrt{x} + C$$

47. $\int \left(\frac{x^3 + x^2 - x + 1}{x^2}\right)dx = \int \left(x + 1 - \frac{1}{x} + \frac{1}{x^2}\right)dx = \frac{1}{2}x^2 + x - \ln|x| - x^{-1} + C$

48. $\int \left(\frac{t^3 + \sqrt[3]{t}}{t^2}\right)dt = \int (t + t^{-5/3})\,dt = \frac{1}{2}t^2 - \frac{3}{2}t^{-2/3} + C$

49. $\int \left(\frac{(x^{1/2} - 1)^2}{x^2}\right)dx = \int \left(\frac{x - 2x^{1/2} + 1}{x^2}\right)dx = \int (x^{-1} - 2x^{-3/2} + x^{-2})\,dx$

$$= \ln|x| + 4x^{-1/2} - x^{-1} + C = \ln|x| + \frac{4}{\sqrt{x}} - \frac{1}{x} + C$$

50. $\int (x+1)^2\left(1-\frac{1}{x}\right)dx = \int (x^2+2x+1)\left(1-\frac{1}{x}\right)dx$

$$= \int \left(x^2+x-1-\frac{1}{x}\right)dx = \frac{1}{3}x^3+\frac{1}{2}x^2-x-\ln|x|+C$$

51. $\int f'(x)\,dx = \int (2x+1)\,dx = x^2+x+C.$ The condition $f(1)=3$ gives $f(1) = 1 + 1 + C = 3$, or $C = 1$. Therefore, $f(x) = x^2 + x + 1$.

52. $f(x) = \int f'(x)\,dx = \int (3x^2-6x)\,dx = x^3-3x^2+C.$ Using the given initial condition, we have $f(2) = 8 - 12 + C = 4$, or $C = 8$. Therefore, $f(x) = x^3 - 3x^2 + 8$.

53. $f'(x) = 3x^2+4x-1;\ f(x) = x^3+2x^2-x+C$. Using the given initial condition, we have $f(2) = 8 + 2(4) - 2 + C = 9$, so $16 - 2 + C = 9$, or $C = -5$. Therefore, $f(x) = x^3 + 2x^2 - x - 5$.

54. $f(x) = \int f'(x)\,dx = \int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = 2x^{1/2}+C.$ Using the given condiition, we obtain $f(4) = 2\sqrt{4}+C = 4+C = 2$, or $C=-2$. Therefore, $f(x) = 2\sqrt{x}-2$.

55. $f(x) = \int f'(x)\,dx = \int \left(1+\frac{1}{x^2}\right)dx = \int (1+x^{-2})\,dx = x-\frac{1}{x}+C.$

Using the given initial condition, we have $f(1) = 1 - 1 + C = 2$, or $C = 2$.

Therefore, $f(x) = x-\frac{1}{x}+2.$

56. $f(x) = \int (e^x-2x)\,dx = e^x-x^2+C.$

Using the initial condition, we have $f(0) = e^0 - 0 + C = 1 + C = 2$, or $C = 1$. So $f(x) = e^x - x^2 + 1$.

57. $f(x) = \int \frac{x+1}{x}\,dx = \int \left(1+\frac{1}{x}\right)dx = x+\ln|x|+C.$ Using the initial condition, we have $f(1) = 1 + \ln 1 + C = 1 + C = 1$, or $C = 0$. So $f(x) = x+\ln|x|$.

58. $f'(x) = 1 + e^x + \frac{1}{x}$; $f(x) = xe^x + \ln|x| + C$

Using the initial condition, we have $f(1) = 1 + e + \ln 1 + C$ and so $3 + e = 1 + e + C$ and $C = 2$. Therefore, $f(x) = xe^x + \ln|x| + 2$.

59. $f(x) = \int f'(x)\,dx = \int \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2}(2x^{1/2}) + C = x^{1/2} + C$; $f(2) = \sqrt{2} + C = \sqrt{2}$ implies $C = 0$. So $f(x) = \sqrt{x}$.

60. $f(t) = \int f'(t)\,dt = \int (t^2 - 2t + 3)\,dt = \frac{1}{3}t^3 - t^2 + 3t + C$

$f(1) = \frac{1}{3} - 1 + 3 + C = 2$ implies $C = -\frac{1}{3}$. So $f(t) = \frac{1}{3}t^3 - t^2 + 3t - \frac{1}{3}$.

61. $f'(x) = e^x + x$; $f(x) = e^x + \frac{1}{2}x^2 + C$; $f(0) = e^0 + \frac{1}{2}(0) + C = 1 + C$

So $3 = 1 + C$ or $2 = C$. Therefore, $f(x) = e^x + \frac{1}{2}x^2 + 2$.

62. $f(x) = \int \left(\frac{2}{x} + 1\right)dx = 2\ln|x| + x + C$. $f(1) = 2\ln 1 + 1 + C = 2$. So $f(x) = 2\ln|x| + x + 1$.

63. The position of the car is

$s(t) = \int f(t)\,dt = \int 2\sqrt{t}\,dt = \int 2t^{1/2}\,dt = 2(\frac{2}{3}t^{3/2}) + C = \frac{4}{3}t^{3/2} + C$.

$s(0) = 0$ implies $s(0) = C = 0$. So $s(t) = \frac{4}{3}t^{3/2}$.

64. Let f be the position function of the maglev. Then $f'(t) = v(t)$. Therefore,

$f(t) = \int f'(t)\,dt = \int v(t)\,dt = \int (0.2t + 3)\,dt = 0.1t^2 + 3t + C$.

If we measure the position of the maglev from the station, then the required function is $f(t) = 0.1t^2 + 3t$.

65. $C(x) = \int C'(x)\,dx = \int (0.000009x^2 - 0.009x + 8)\,dx$

$= 0.000003x^3 - 0.0045x^2 + 8x + k$.

$C(0) = k = 120$ and so $C(x) = 0.000003x^3 - 0.0045x^2 + 8x + 120$.

$C(500) = 0.000003(500)^3 - 0.0045(500)^2 + 8(500) + 120$, or \$3370.

66. a. $R(x) = \int R'(x)\,dx = \int (-0.009x + 12)\,dx = -0.0045x^2 + 12x + C$. But

$R(0) = C = 0$ and so $R(x) = -0.0045x^2 + 12x$

b. $R(x) = px$ and so $-0.0045x^2 + 12x = px$ or $p = -0.0045x + 12$.

67. $P'(x) = -0.004x + 20$, $P(x) = -0.002x^2 + 20x + C$. Since $C = -16{,}000$, we find that $P(x) = -0.002x^2 + 20x - 16{,}000$. The company realizes a maximum profit when $P'(x) = 0$, that is, when $x = 5000$ units. Next,

$$P(5000) = -0.002(5000)^2 + 20(5000) - 16{,}000 = 34{,}000.$$

Thus, a maximum profit of \$34,000 is realized at a production level of 5000 units.

68. $C(x) = \int C'(x)\,dx = \int (0.002x + 100)\,dx = 0.001x^2 + 100x + k$. But

$C(0) = k = 4000$ and so $C(x) = 0.001x^2 + 100x + 4000$.

69. a. $N(t) = \int N'(t)\,dt = \int (-3t^2 + 12t + 45)\,dt = -t^3 + 6t^2 + 45t + C$. But $N(0) = C = 0$

and so $N(t) = -t^3 + 6t^2 + 45t$.

b. The number is $N(4) = -4^3 + 6(4)^2 + 45(4) = 212$.

70. a. We have the initial-value problem: $T'(t) = 0.15t^3 - 3.6t + 14.4$; $T(0) = 24$

Integrating, we find

$$T(t) = \int T'(t)dt = \int (0.15t^3 - 3.6t + 14.4)dt = 0.05t^3 - 1.8t^2 + 14.4t + C$$

Using the initial condition, we find $T(0) = 24 = 0 + C$, so $C = 24$.

Therefore, $T(t) = 0.05t^3 - 1.8t^2 + 14.4t + 24$.

b. The temperature at 10 A.M. was

$$T(4) = 0.05(4^3) - 1.8(4^2) + 14.4(4) + 24 = 56 \text{ or } 56^\circ\text{F}.$$

71. a. We have the initial-value problem:

$$C'(t) = 12.288t^2 - 150.5594t + 695.23$$
$$C(0) = 3142$$

Integrating, we find

$$C(t) = \int C'(t)dt = \int (12.288t^2 - 150.5594t + 695.23)dt$$
$$= 4.096t^3 - 75.2797t^2 + 695.23t + k$$

Using the initial condition, we find

$$C(0) = 0 + k = 3142, \quad \text{and so } k = 3142.$$

Therefore, $C(t) = 4.096t^3 - 75.2797t^2 + 695.23t + 3142$.

The projected average out-of-pocket costs for beneficiaries is 2010 is

$$C(2) = 4.096(8) - 75.2797(4) + 695.23(2) + 3142 = 4264.1092$$

or \$4264.11.

72. a. $h(t) = \int h'(t)\,dt = \int -32t\,dt = -16t^2 + C$. But $h(0) = C = 400$ and so

$h(t) = -16t^2 + 400$.

b. It strikes the ground when $h(t) = 0$; that is, when $-16t^2 + 400 = 0$, or $t = 5$.

c. Its velocity is $-32(5)$ or 160 ft/sec downwards.

73. The number of new subscribers at any time is

$$N(t) = \int (100 + 210t^{3/4})\,dt = 100t + 120t^{7/4} + C.$$

The given condition implies that $N(0) = 5000$. Using this condition, we find $C = 5000$. Therefore, $N(t) = 100t + 120t^{7/4} + 5000$. The number of subscribers 16 months from now is

$$N(16) = 100(16) + 120(16)^{7/4} + 5000, \text{ or } 21{,}960.$$

EXERCISES 6.2, page 419

1. Put $u = 4x + 3$ so that $du = 4\,dx$, or $dx = \frac{1}{4}du$. Then

$$\int 4(4x+3)^4\,dx = \int u^4\,du = \tfrac{1}{5}u^5 + C = \tfrac{1}{5}(4x+3)^5 + C.$$

2. Let $u = 2x^2 + 1$ so that $du = 4x\,dx$. Then

$$\int 4x(2x^2+1)^7\,dx = \int u^7\,du = \tfrac{1}{8}u^8 + C = \tfrac{1}{8}(2x^2+1)^8 + C.$$

3. Let $u = x^3 - 2x$ so that $du = (3x^2 - 2)\,dx$. Then

$$\int (x^3-2x)^2(3x^2-2)\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}(x^3-2x)^3 + C.$$

4. Put $u = x^3 - x^2 + x$ so that $du = (3x^2 - 2x + 1)\,dx$. Then,

$$\int (3x^2-2x+1)(x^3-x^2+x)^4\,dx = \int u^4\,du = \tfrac{1}{5}u^5 + C = \tfrac{1}{5}(x^3-x^2+x)^5 + C$$

5. Let $u = 2x^2 + 3$ so that $du = 4x\,dx$. Then

$$\int \frac{4x}{(2x^2+3)^3}\,dx = \int \frac{1}{u^3}\,du = \int u^{-3}\,du = -\tfrac{1}{2}u^{-2} + C = -\frac{1}{2(2x^2+3)^2} + C.$$

6. Let $u = x^3 + 2x$ so that $du = (3x^2 + 2)\,dx$. Then

$$\int \frac{3x^2+2}{(x^3+2x)^2}\,dx = \int \frac{du}{u^2} = \int u^{-2}\,du = -u^{-1} + C = -\frac{1}{x^3+2x} + C.$$

7. Put $u = t^3 + 2$ so that $du = 3t^2\,dt$ or $t^2\,dt = \frac{1}{3}du$. Then

$$\int 3t^2\sqrt{t^3+2}\,dt = \int u^{1/2}\,du = \tfrac{2}{3}u^{3/2} + C = \tfrac{2}{3}(t^3+2)^{3/2} + C$$

8. Let $u = t^3 + 2$ so that $du = 3t^2\,dt$. Then

$$\int 3t^2(t^3+2)^{3/2}\,dt = \int u^{3/2}\,du = \tfrac{2}{5}u^{5/2} + C = \tfrac{2}{5}(t^3+2)^{5/2} + C.$$

9. Let $u = x^2 - 1$ so that $du = 2x\,dx$ and $x\,dx = \frac{1}{2}du$. Then,

$$\int (x^2-1)^9\,x\,dx = \int \tfrac{1}{2}u^9\,du = \tfrac{1}{20}u^{10} + C = \tfrac{1}{20}(x^2-1)^{10} + C.$$

10. Let $u = 2x^3 + 3$ so that $du = 6x^2\,dx$ or $x^2\,dx = \frac{1}{6}du$. Then

$$\int x^2(2x^3+3)^4\,dx = \tfrac{1}{6}\int u^4\,du = \tfrac{1}{30}u^5 + C = \tfrac{1}{30}(2x^3+3)^5 + C.$$

11. Let $u = 1 - x^5$ so that $du = -5x^4\,dx$ or $x^4\,dx = -\frac{1}{5}du$. Then

$$\int \frac{x^4}{1-x^5}\,dx = -\frac{1}{5}\int \frac{du}{u} = -\frac{1}{5}\ln|u| + C = -\frac{1}{5}\ln\left|1-x^5\right| + C.$$

12. Let $u = x^3 - 1$ so that $du = 3x^2\,dx$ or $x^2\,dx = \frac{1}{3}du$. Then

$$\int \frac{x^2}{\sqrt{x^3-1}}\,dx = \frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int u^{-1/2}\,du = \frac{2}{3}u^{1/2} + C = \frac{2}{3}\sqrt{x^3-1} + C.$$

13. Let $u = x - 2$ so that $du = dx$. Then

$$\int \frac{2}{x-2}\,dx = 2\int \frac{du}{u} = 2\ln|u| + C = \ln u^2 + C = \ln(x-2)^2 + C$$

14. Let $u = x^3 - 3$, so that $du = 3x^2\,dx$, and $\frac{1}{3}du = x^2\,dx$.

$$\int \frac{x^2}{x^3-3}\,dx = \int \frac{du}{3u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln\left|x^3-3\right| + C.$$

15. Let $u = 0.3x^2 - 0.4x + 2$. Then $du = (0.6x - 0.4)\,dx = 2(0.3x - 0.2)\,dx$.

$$\int \frac{0.3x-0.2}{0.3x^2-0.4x+2}\,dx = \int \frac{1}{2u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(0.3x^2-0.4x+2) + C.$$

16. Let $u = 0.2x^3 + 0.3x$. Then $du = (0.6x^2 + 0.3)dx = 0.3(2x^2 + 1)\,dx$.

$$\int \frac{2x^2+1}{0.2x^3+0.3x}\,dx = \int \frac{1}{0.3u}\,du = \frac{1}{0.3}\ln|u| + C = \frac{10}{3}\ln|0.2x^3+0.3x| + C.$$

17. Let $u = 3x^2 - 1$ so that $du = 6x\,dx$, or $x\,dx = \frac{1}{6}\,du$. Then

$$\int \frac{x}{3x^2-1}\,dx = \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|3x^2-1| + C.$$

18. $I = \int \frac{x^2-1}{x^3-3x+1}\,dx$. Let $u = x^3 - 3x + 1$. Then, $du = (3x^2 - 3)\,dx = 3(x^2 - 1)\,dx$,

or $(x^2 - 1)\,dx = \frac{1}{3}\,du$. Therefore,

$$I = \int \frac{1}{3}u^{-1}\,du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-3x+1| + C$$

19. Let $u = -2x$ so that $du = -2\,dx$ or $dx = -\frac{1}{2}du$. Then

$$\int e^{-2x}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-2x} + C.$$

20. Let $u = -0.02x$ so that $du = -0.02\,dx$ or $dx = -\frac{1}{0.02}\,du = -50\,du$. Then

$$\int e^{-0.02x}\,dx = -50\int e^u\,du = -50e^{-0.02x} + C.$$

21. Let $u = 2 - x$ so that $du = -\,dx$ or $dx = -\,du$. Then

$$\int e^{2-x}dx = -\int e^u\,du = -e^u + C = -e^{2-x} + C.$$

22. Let $u = 2t + 3$ so that $du = 2\,dt$ or $dt = \frac{1}{2}du$.

$$\int e^{2t+3}\,dt = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{2t+3} + C.$$

23. Let $u = -x^2$, then $du = -2x\,dx$ or $x\,dx = -\frac{1}{2}du$.

$$\int xe^{-x^2}\,dx = \int -\tfrac{1}{2}e^u\,du = -\tfrac{1}{2}e^u + C = -\tfrac{1}{2}e^{-x^2} + C.$$

24. Let $u = x^3 - 1$, so that $du = 3x^2\,dx$ and $x^2\,dx = \frac{1}{3}du$. Then

$$\int x^2 e^{x^3-1}\,dx = \int \tfrac{1}{3}e^u\,du = \tfrac{1}{3}e^u + C = \tfrac{1}{3}e^{x^3-1} + C.$$

25. $\int (e^x - e^{-x})\,dx = \int e^x\,dx - \int e^{-x}\,dx = e^x - \int e^{-x}\,dx.$

To evaluate the second integral on the right, let $u = -x$ so that $du = -dx$ or $dx = -du$. Therefore,

$$\int (e^x - e^{-x})\,dx = e^x + \int e^u\,du = e^x + e^u + C = e^x + e^{-x} + C.$$

26. $\int (e^{2x} + e^{-3x})\,dx = \int e^{2x}\,dx + \int e^{-3x}\,dx$. To evaluate the first integral, let $u = 2x$, and to evaluate the second, let $u = -3x$. We find

$$\int (e^{2x} + e^{-3x})\,dx = \tfrac{1}{2}e^{2x} - \tfrac{1}{3}e^{-3x} + C.$$

27. Let $u = 1 + e^x$ so that $du = e^x\,dx$. Then

$$\int \frac{e^x}{1+e^x}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(1+e^x) + C.$$

28. Let $u = 1 + e^{2x}$ so that $du = 2e^{2x}\,dx$. Then $e^{2x}\,dx = \frac{1}{2}\,du$.

$$\int \frac{e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(1+e^{2x}) + C.$$

29. Let $u = \sqrt{x} = x^{1/2}$. Then $du = \frac{1}{2}x^{-1/2}\,dx$ or $2\,du = x^{-1/2}\,dx$.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 2e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C.$$

30. Let $u = e^{-1/x}$, then $du = -\dfrac{1}{x^2}e^{-1/2}\,dx.$

$$\int \frac{e^{-1/x}}{x^2}\,dx = \int -u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}e^{-2/x} + C.$$

31. Let $u = e^{3x} + x^3$ so that $du = (3e^{3x} + 3x^2)\,dx = 3(e^{3x} + x^2)\,dx$ or $(e^{3x} + x^2)\,dx = \frac{1}{3}du$. Then

$$\int \frac{e^{3x} + x^2}{(e^{3x} + x^3)^3}\,dx = \frac{1}{3}\int \frac{du}{u^3} = \frac{1}{3}\int u^{-3}\,du = -\frac{1}{6}u^{-2} + C = -\frac{1}{6(e^{3x} + x^3)^2} + C.$$

32. Let $u = e^x + e^{-x}$, so that $du = e^x - e^{-x}\,dx$.

$$\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^{3/2}}\,dx = \int \frac{du}{u^{3/2}} = \int u^{-3/2}\,du = -2u^{-1/2} + C = -2(e^x + e^{-x})^{-1/2} + C.$$

33. Let $u = e^{2x} + 1$, so that $du = 2e^{2x}\,dx$, or $\frac{1}{2}du = e^{2x}\,dx$.

$$\int e^{2x}(e^{2x} + 1)^3\,dx = \int \tfrac{1}{2}u^3\,du = \tfrac{1}{8}u^4 + C = \tfrac{1}{8}(e^{2x} + 1)^4 + C.$$

34. Let $u = 1 + e^{-x}\,dx$ so that $du = -e^{-x}\,dx$.

$$\int e^{-x}(1 + e^{-x})\,dx = \int -u\,du = -\tfrac{1}{2}u^2 + C = -\tfrac{1}{2}(1 + e^{-x})^2 + C.$$

35. Let $u = \ln 5x$ so that $du = \dfrac{1}{x}\,dx$. Then

$$\int \frac{\ln 5x}{x}\,dx = \int u\,du = \tfrac{1}{2}u^2 + C = \tfrac{1}{2}(\ln 5x)^2 + C.$$

36. Let $v = \ln u$ so that $dv = \frac{1}{u}du$. Then

$$\int \frac{(\ln u)^3}{u}\,du = \int v^3\,dv = \tfrac{1}{4}v^4 + C = \tfrac{1}{4}(\ln u)^4 + C.$$

37. Let $u = \ln x$ so that $du = \frac{1}{x}dx$. Then

$$\int \frac{1}{x\ln x}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C.$$

38. Let $u = \ln x$ so that $du = \frac{1}{x}\,dx$. Then

$$\int \frac{1}{x(\ln x)^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} + C = -\frac{1}{\ln x} + C.$$

39. Let $u = \ln x$ so that $du = \frac{1}{x}\,dx$. Then

$$\int \frac{\sqrt{\ln x}}{x}\,dx = \int \sqrt{u}\,du = \tfrac{2}{3}u^{3/2} + C = \tfrac{2}{3}(\ln x)^{3/2} + C$$

40. Let $u = \ln x$, so that $du = \frac{1}{x}\,dx$. Then

$$\int \frac{(\ln x)^{7/2}}{x}\,dx = \int u^{7/2}\,du = \tfrac{2}{9}u^{9/2} + C = \tfrac{2}{9}(\ln x)^{3/2} + C.$$

41. $$\int \left(xe^{x^2} - \frac{x}{x^2+2} \right) dx = \int xe^{x^2} - \int \frac{x}{x^2+2}\,dx.$$

To evaluate the first integral, let $u = x^2$ so that $du = 2x\,dx$, or $x\,dx = \frac{1}{2}\,du$. Then

$$\int xe^{x^2}\,dx = \tfrac{1}{2}\int e^u\,du + C_1 = \tfrac{1}{2}e^u + C_1 = \tfrac{1}{2}e^{x^2} + C_1.$$

To evaluate the second integral, let $u = x^2 + 2$ so that $du = 2x\,dx$, or $x\,dx = \frac{1}{2}\,du$. Then

$$\int \frac{x}{x^2+2}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C_2 = \frac{1}{2}\ln(x^2+2) + C_2.$$

Therefore, $\displaystyle\int \left(xe^{x^2} - \frac{x}{x^2+2} \right) dx = \frac{1}{2}e^{x^2} - \frac{1}{2}\ln(x^2+2) + C.$

42. $$\int \left(xe^{-x^2} + \frac{e^x}{e^x+3} \right) dx = \int xe^{-x^2}\,dx + \int \frac{e^x}{e^x+3}\,dx.$$

To evaluate the first integral, let $u = -x^2$ so that $du = -2x\,dx$, or $x\,dx = -\frac{1}{2}\,du$. Then

$$\int xe^{-x^2}\,dx = -\tfrac{1}{2}\int e^u\,du = -\tfrac{1}{2}e^u + C_1 = -\tfrac{1}{2}e^{-x^2} + C_1.$$

To evaluate the second integral, let $u = e^x + 3$ so that $du = e^x\,dx$. Then

$$\int \frac{e^x}{e^x+3}\,dx = \int \frac{du}{u} = \ln|u| + C_2 = \ln(e^x+3) + C_2.$$

Therefore, $\displaystyle\int \left(xe^{-x^2} - \frac{e^x}{e^x+3} \right) dx = -\frac{1}{2}e^{-x^2} + \ln(e^x+3) + C.$

43. Let $u = \sqrt{x} - 1$ so that $du = \frac{1}{2}x^{-1/2}\,dx = \dfrac{1}{2\sqrt{x}}\,dx$ or $dx = 2\sqrt{x}\,du$.

Also, we have $\sqrt{x} = u + 1$, so that $x = (u+1)^2 = u^2 + 2u + 1$ and $dx = 2(u+1)\,du$. So

$$\int \frac{x+1}{\sqrt{x}-1}\,dx = \int \frac{u^2+2u+2}{u}\cdot 2(u+1)\,du = 2\int \frac{(u^3+3u^2+4u+2)}{u}\,du$$

$$= 2\int \left(u^2 + 3u + 4 + \frac{2}{u} \right) du = 2\left(\frac{1}{3}u^3 + \frac{3}{2}u^2 + 4u + 2\ \ln|u| \right) + C$$

$$= 2\left[\frac{1}{3}(\sqrt{x}-1)^3 + \frac{3}{2}(\sqrt{x}-1)^2 + 4(\sqrt{x}-1) + 2\ \ln\left|\sqrt{x}-1\right| \right] + C.$$

44. Let $v = e^{-u} + u$. Then $dv = (-e^{-u} + 1)\,du$, or $-dv = (e^{-u} - 1)\,du$.

Therefore, $\displaystyle\int \frac{e^{-u}-1}{e^{-u}+u}\,du = \int -\frac{dv}{v} = -\ln|v| = -\ln\left|e^{-u}+u\right| + C.$

45. Let $u = x - 1$ so that $du = dx$. Also, $x = u + 1$ and so

$$\int x(x-1)^5\,dx = \int (u+1)u^5\,du = \int (u^6+u^5)\,du$$

$$= \frac{1}{7}u^7 + \frac{1}{6}u^6 + C = \frac{1}{7}(x-1)^7 + \frac{1}{6}(x-1)^6 + C$$

$$= \frac{(6x+1)(x-1)^6}{42} + C.$$

46. $\displaystyle\int \frac{t}{t+1}\,dt = \int \left(1 - \frac{1}{t+1}\right)dt = \int dt - \int \frac{1}{t+1}\,dt = t - \ln|t+1| + C.$

47. Let $u = 1 + \sqrt{x}$ so that $du = \frac{1}{2}x^{-1/2}\,dx$ and $dx = 2\sqrt{x} = 2(u-1)\,du$

$$\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx = \int \left(\frac{1-(u-1)}{u}\right)\cdot 2(u-1)\,du = 2\int \frac{(2-u)(u-1)}{u}\,du$$

$$= 2\int \frac{-u^2+3u-2}{u}\,du = 2\int \left(-u+3-\frac{2}{u}\right)du = -u^2+6u-4\ln|u|+C$$

$$= -(1+\sqrt{x})^2 + 6(1+\sqrt{x}) - 4\ln(1+\sqrt{x}) + C$$

$$= -1-2\sqrt{x}-x+6+6\sqrt{x}-4\ln(1+\sqrt{x})+C$$

$$= -x+4\sqrt{x}+5-4\ln(1+\sqrt{x})+C.$$

48. Let $u = 1-\sqrt{x}$ so that $du = -\dfrac{1}{2\sqrt{x}}\,dx$ and $dx = -2\sqrt{x}\,du$.

Then $\sqrt{x} = 1-u$ and $dx = -2(1-u)\,du$. So

$$\int \frac{1+\sqrt{x}}{1-\sqrt{x}}\,dx = \int \left(\frac{2-u}{u}\right)(-2)(1-u)\,du = -2\int \frac{(u-2)(u-1)}{u}\,du$$

$$= -2\int \frac{u^2-3u+2}{u}\,du = -2\int \left(u-3+\frac{2}{u}\right)du$$

$$= -2\left(\frac{1}{2}u^2-3u+2\ln|u|\right)+C = 6u-u^2-4\ln|u|+C$$

$$= 6(1-\sqrt{x})-(1-\sqrt{x})^2-4\ln(1-\sqrt{x})+C.$$

49. $I = \int v^2(1-v)^6\,dv$. Let $u = 1-v$, then $du = -dv$. Also, $1-u = v$, and $(1-u)^2 = v^2$. Therefore,

$$I = \int -(1-2u+u^2)u^6\,du = \int -(u^6-2u^7+u^8)\,du = -\left(\frac{u^7}{7}-\frac{2u^8}{8}+\frac{u^9}{9}\right)+C$$

$$= -u^7\left(\frac{1}{7}-\frac{1}{4}u+\frac{1}{9}u^2\right)+C = -\frac{1}{252}(1-v)^7[36-63(1-v)+28(1-2v+v^2)]$$

$$= -\frac{1}{252}(1-v)^7[36-63+63v+28-56v+28v^2]$$

$$= -\frac{1}{252}(1-v)^7(28v^2+7v+1)+C.$$

50. Let $u = x^2 + 1$ so that $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. Then

$$\int x^3(x^2+1)^{3/2}\,dx = \int x^2(x^2+1)^{3/2}\,x\,dx$$

$$= \int (u-1)u^{3/2}\,\tfrac{1}{2}\,du \qquad (x^2 = u-1)$$

$$= \frac{1}{2}\int (u^{5/2} - u^{3/2})\,du = \frac{1}{2}\left(\frac{2}{7}u^{7/2} - \frac{2}{5}u^{5/2}\right) + C$$

$$= \frac{u^{5/2}}{35}(5u-7) + C = \frac{1}{35}(x^2+1)^{5/2}(5x^2-2) + C.$$

51. $f(x) = \int f'(x)\,dx = 5\int (2x-1)^4\,dx$. Let $u = 2x - 1$ so that $du = 2x\text{-}1$ so that $du = 2\ dx$, or $dx = \frac{1}{2}\,du$. Then

$$f(x) = \frac{5}{2}\int u^4\,du = \frac{1}{2}u^5 + C = \frac{1}{2}(2x-1)^5 + C.$$

Next, $f(1) = 3$ implies $\frac{1}{2} + C = 3$ or $C = \frac{5}{2}$. Therefore,

$$f(x) = \frac{1}{2}(2x-1)^5 + \frac{5}{2}.$$

52. $f(x) = \int f'(x)\,dx = \int \frac{3x^2}{2\sqrt{x^3-1}}\,dx$. Let $u = (x^3 - 1)$ so that $du = 3x^2\,dx$.

Then $f(x) = \int \frac{du}{2\sqrt{u}} = \frac{1}{2}\int u^{-1/2}\,du = \left(\frac{1}{2}\right)2u^{1/2} + C = u^{1/2} + C = (x^3-1)^{1/2} + C.$

Next, $f(1) = (0) + C = 1.$ Therefore, C = 1. Hence $f(x) = \sqrt{x^3+1} + 1.$

53. $f(x) = \int -2xe^{-x^2+1}\,dx$. Let $u = -x^2 + 1$ so that $du = -2x\,dx$. Then

$f(x) = \int e^u\,du = e^u + C = e^{-x^2+1} + C$. The condition $f(1) = 0$ implies

$f(1) = 1 + C = 0$, or $C = -1$. Therefore, $f(x) = e^{-x^2+1} - 1.$

54. $f(x) = \int f'(x)\,dx = \int \left(1 - \frac{2x}{x^2+1}\right)dx = \int dx - \int \frac{2x}{x^2+1}\,dx.$

Let us make the substitution $u = x^2 + 1$ for the second integral on the right. With $du = 2x\,dx$, we find

$$f(x) = \int dx - \int \frac{du}{u} = x - \ln|u| + C = x - \ln(x^2 + 1) + C.$$

The condition that the graph of f passes through (0,2) translates into the condition $f(0) = 2$. Using this condition, we find $f(0) = C = 2$. Therefore, the required function is $f(x) = x - \ln(x^2 + 1) + 2$.

55. $N'(t) = 2000(1 + 0.2t)^{-3/2}$. Let $u = 1 + 0.2t$. Then $du = 0.2\,dt$ and $5\,du = dt$. Therefore, $N(t) = (5)(2000)$

$$\int u^{-3/2}\,du = -20{,}000u^{-1/2} + C = -20{,}000(1 + 0.2t)^{-1/2} + C.$$

Next, $N(0) = -20{,}000(1)^{-1/2} + C = 1000$. Therefore, $C = 21{,}000$ and $N(t) = -\dfrac{20{,}000}{\sqrt{1+0.2t}} + 21{,}000$. In particular, $N(5) = -\dfrac{20{,}000}{\sqrt{2}} + 21{,}000 \approx 6{,}858$.

56. The number of viewers in the tth year is given by $N(t) = \int 3(2 + \frac{1}{2}t)^{-1/3}\,dt$.

To evaluate the integral, let $u = 2 + \frac{1}{2}t$ so that $du = \frac{1}{2}\,dt$ and $dt = 2\,du$. Then

$$N(t) = 6\int u^{-1/3}\,du = 9u^{2/3} + C = 9(2 + \tfrac{1}{2}t)^{2/3} + C.$$

The given condition implies that $N(1) = 9(\frac{5}{2})^{2/3} + C$. Using this condition, we see that $N(1) = 9(\frac{5}{2})^{2/3} + C = 9(\frac{5}{2})^{2/3}$ so that $C = 0$. Therefore, $N(t) = 9(2 + \frac{1}{2}t)^{2/3}$. The number of viewers in the 2000 season is given by $N(5) = 9(5)^{2/3} \approx 26.32$, or approximately 26.3 million viewers.

57. $p(x) = \displaystyle\int -\frac{250x}{(16 + x^2)^{3/2}}\,dx = -250\int \frac{x}{(16 + x^2)^{3/2}}\,dx.$

Let $u = 16 + x^2$ so that $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$.

Then $p(x) = -\frac{250}{2}\int u^{-3/2}\,du = (-125)(-2)u^{-1/2} + C = \dfrac{250}{\sqrt{16 + x^2}} + C.$

$p(3) = \dfrac{250}{\sqrt{16+9}} + C = 50$ implies $C = 0$ and $p(x) = \dfrac{250}{\sqrt{16 + x^2}}$.

58 Let $u = (5 - x)$ so that $du = -x\,dx$. Then

$$p(x) = \int \frac{240}{(5-x)^2}\,dx = 240\int (5-x)^{-2}\,dx = 240\int -u^{-2}\,du = 240u^{-1} + C = \frac{240}{5-x} + C.$$

Next, the condition $p(2) = 50$ gives $\frac{240}{3} + C = 80 + C = 50$, or $C = 30$. Therefore,

$$p(x) = \frac{240}{5-x} + 30.$$

59. Let $u = 2t + 4$, so that $du = 2\,dt$. Then

$$r(t) = \int \frac{30}{\sqrt{2t+4}}\,dt = 30\int \frac{1}{2}u^{-1/2}\,du = 30u^{1/2} + C = 30\sqrt{2t+4} + C.$$

$r(0) = 60 + C = 0$, and $C = -60$. Therefore, $r(t) = 30\left(\sqrt{2t+4} - 2\right)$. Then

$r(16) = 30\left(\sqrt{36} - 2\right) = 120$ ft. Therefore, the polluted area is

$$\pi r^2 = \pi(120)^2 = 14{,}400\pi, \qquad \text{or } 14{,}400\pi \text{ sq ft.}$$

60. Let $u = 1 + 1.09t$, then $du = 1.09\,dt$. So

$$\int \frac{5.45218}{(1+1.09t)^{0.9}}\,dt = 5.45218\int (1+1.09t)^{-0.9}\,dt = \frac{5.45218}{1.09}\int u^{-0.9}\,du$$
$$= 50.02\,u^{0.1} + C = 50.02(1+1.09t)^{0.1} + C.$$

Then $g(0) = 50.02 + C = 50.02$ and $C = 0$. So $g(t) = 50.02(1 + 1.09t)^{0.1}$ and $g(100) = 50.02(110)^{0.1} \approx 80.04$.

61. The population t years from now will be

$$P(t) = \int r(t)dt = \int 400\left(1 + \frac{2t}{24+t^2}\right)dt = \int 400dt + 800\int \frac{t}{24+t^2}\,dt$$

In order to evaluate the second integral on the right, let

$$u = 24 + t^2, \quad du = 2t\,dt, \quad or \quad t\,dt = \tfrac{1}{2}du$$

We obtain $P(t) = 400t + 800\int \frac{\frac{1}{2}du}{u} = 400t + 400\left|\ln u\right| + C$

$$= 400[t + \ln(24 + t^2)] + C$$

To find C, use the condition $P(0) = 60{,}000$ giving

$$400[0 + \ln 24] + C = 60{,}000 \quad \text{or} \quad C = 58728.78$$

So $P(t) = 400[t + \ln(24 + t^2)] + 58728.78$. Therefore, the population 5 years from now will be

$$400[5 + \ln(24 + 25)] + 58728.78 \approx 62{,}285.51, \text{ or approximately, } 62{,}286.$$