MATH180 – HOMEWORK SOLUTIONS

HOMEWORK #6

Section 5.2: 1-43 (odd only), 49, 53

Section 5.3: 3, 5, 7, 13, 19, 25, 37

Section 5.4: 1-39 (odd only), 43, 45, 47, 70

Section 5.5: 1-35 (odd only), 39, 41, 45, 51, 55

EXERCISES 5.2 , page 346

1. $\log_2 64 = 6$
2. $\log_3 243 = 5$
3. $\log_3 \frac{1}{9} = -2$
4. $\log_5 \frac{1}{125} = -3$
5. $\log_{1/3} \frac{1}{3} = 1$
6. $\log_{1/2} 16 = -4$
7. $\log_{32} 8 = \frac{3}{5}$
8. $\log_{81} 27 = \frac{3}{4}$
9. $\log_{10} 0.001 = -3$
10. $\log_{16} 0.5 = -\frac{1}{4}$.

11. $\log 12 = \log 4 \times 3 = \log 4 + \log 3 = 0.6021 + 0.4771 = 1.0792$.

12. $\log \frac{3}{4} = \log 3 - \log 4 = 0.4771 - 0.6021 = -0.125$.

13. $\log 16 = \log 4^2 = 2 \log 4 = 2(0.6021) = 1.2042$.

14. $\log \sqrt{3} = \log 3^{1/2} = \frac{1}{2} \log 3 = \frac{1}{2}(0.4771) = 0.2386$.

15. $\log 48 = \log 3 \times 4^2 = \log 3 + 2 \log 4 = 0.4771 + 2(0.6021) = 1.6813$.

16. $\log \frac{1}{300} = \log 1 - \log 300 = -\log 300 = -\log (3 \times 100)$
$= -(\log 3 + \log 100) = -(\log 3 + 2 \log 10) = -(\log 3 + 2) = -2.4771$.

17. $2\ln a + 3\ln b = \ln a^2 b^3$.

18. $\frac{1}{2}\ln x + 2\ln y - 3\ln z = \ln \frac{x^{1/2} y^2}{3z} = \ln \frac{\sqrt{x} y^2}{3z}$

19. $\ln 3 + \frac{1}{2}\ln x + \ln y - \frac{1}{3}\ln z = \ln \frac{3\sqrt{x} y}{\sqrt[3]{z}}$

20. $\ln 2 + \frac{1}{2}\ln(x+1) - 2\ln(1+\sqrt{x}) = \ln \frac{2(x+1)^{1/2}}{(1+\sqrt{x})^2}$

21. $\log x(x+1)^4 = \log x + \log (x+1)^4 = \log x + 4 \log (x+1)$.

22. $\log x(x^2+1)^{-1/2} = \log x - \frac{1}{2} \log (x^2+1)$.

23. $\log \dfrac{\sqrt{x+1}}{x^2+1} = \log (x+1)^{1/2} - \log(x^2+1) = \frac{1}{2} \log (x+1) - \log (x^2+1)$

24. $\ln \dfrac{e^x}{1+e^x} = x - \ln (1+e^x)$.

25. $\ln xe^{-x^2} = \ln x - x^2$.

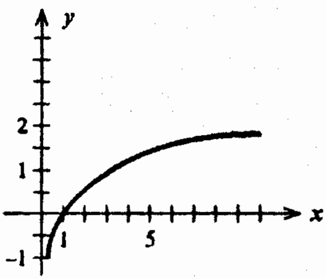
26. $\ln x(x+1)(x+2) = \ln x + \ln (x+1) + \ln (x+2)$.

27. $\ln \left(\dfrac{x^{1/2}}{x^2\sqrt{1+x^2}} \right) = \ln x^{1/2} - \ln x^2 - \ln (1+x^2)^{1/2}$

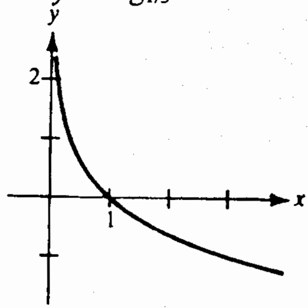
$= \frac{1}{2} \ln x - 2 \ln x - \frac{1}{2} \ln (1+x^2) = -\frac{3}{2} \ln x - \frac{1}{2} \ln (1+x^2)$.

28. $\ln \dfrac{x^2}{\sqrt{x}(1+x)^2} = 2 \ln x - \frac{1}{2} \ln x - 2 \ln (1+x) = \frac{3}{2} \ln x - 2 \ln (1+x)$.

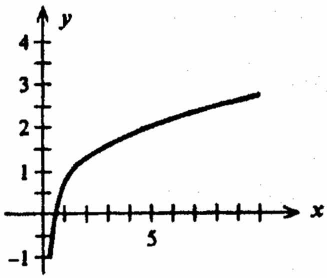
29. $y = \log_3 x$



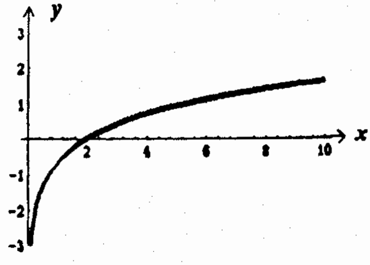
30. $y = \log_{1/3} x$



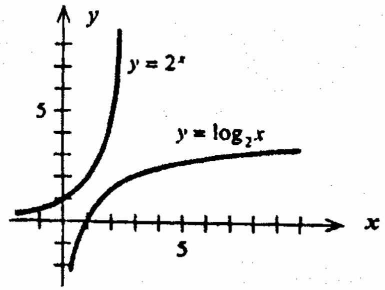
31. $y = \ln 2x$



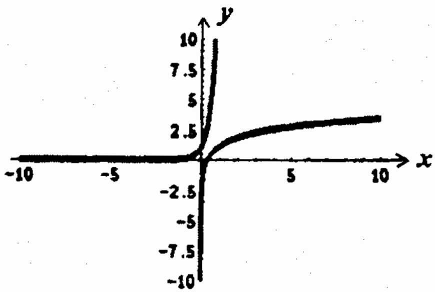
32. $y = \ln \frac{1}{2} x$



33. $y = 2^x$ and $y = \log_2 x$



34. $y = e^{3x}$ and $y = \ln 3x$



35. $e^{0.4t} = 8$, $0.4t \ln e = \ln 8$, and $0.4t = \ln 8$ ($\ln e = 1$.) So, $t = \dfrac{\ln 8}{0.4} = 5.1986$.

36. $\frac{1}{3}e^{-3t} = 0.9$, $e^{-3t} = 2.7$. Taking the logarithm, we have

$$-3t \ln e = \ln 2.7,\ t = -\frac{\ln 2.7}{3} \approx -0.3311.$$

37. $5e^{-2t} = 6$, $e^{-2t} = \frac{6}{5} = 1.2$. Taking the logarithm, we have

$$-2t \ln e = \ln 1.2, \text{ or } t = -\frac{\ln 1.2}{2} \approx -0.0912.$$

38. $4e^{t-1} = 4$, $e^{t-1} = 1$, $\ln e^{t-1} = \ln 1$, $(t-1)\ln e = 0$, $t = 1$.

39. $2e^{-0.2t} - 4 = 6$, $2e^{-0.2t} = 10$. Taking the logarithm on both sides of this last equation, we have $\ln e^{-0.2t} = \ln 5$; $-0.2t \ln e = \ln 5$; $-0.2t = \ln 5$;

and $\quad t = -\dfrac{\ln 5}{0.2} \approx -8.0472$.

40. $12 - e^{0.4t} = 3$, $e^{0.4t} = 9$, $\ln e^{0.4t} = \ln 9$, $0.4t \ln e = \ln 9$, $0.4t = \ln 9$,

so $t = \dfrac{\ln 9}{0.4} \approx 5.4931$.

41. $\dfrac{50}{1+4e^{0.2t}} = 20$, $1 + 4e^{0.2t} = \dfrac{50}{20} = 2.5$, $4e^{0.2t} = 1.5$,

$e^{0.2t} = \dfrac{1.5}{4} = 0.375$, $\ln e^{0.2t} = \ln 0.375$, $0.2t = \ln 0.375$. So $t = \dfrac{\ln 0.375}{0.2} \approx -4.9041$.

42. $\dfrac{200}{1+3e^{-0.3t}} = 100$, $1 + 3e^{-0.3t} = \dfrac{200}{100} = 2$, $3e^{-0.3t} = 1$; $e^{-0.3t} = \dfrac{1}{3}$,

$\ln e^{-0.3t} = \ln \frac{1}{3} = \ln 1 - \ln 3 = -\ln 3$. So $-0.3t \ln e = -\ln 3$, or $0.3t = \ln 3$.

So $t = \frac{\ln 3}{0.3} \approx 3.6620$.

43. Taking the logarithm on both sides, we obtain

$\ln A = \ln Be^{-t/2}$, $\ln A = \ln B + \ln e^{-t/2}$, $\ln A - \ln B = -t/2 \ln e$,

$\ln \frac{A}{B} = -\frac{t}{2}$ or $t = -2 \ln \frac{A}{B} = 2 \ln \frac{B}{A}$

44. $\frac{A}{1+Be^{t/2}} = C$, $A = C + BCe^{t/2}$, $A - C = BCe^{t/2}$;

$\frac{A-C}{BC} = e^{t/2}$, $\frac{t}{2} = \ln \frac{A-C}{BC}$, $t = 2 \ln\left(\frac{A-C}{BC}\right)$.

45. $p(x) = 19.4 \ln x + 18$. For a child weighing 92 lb, we find
$p(92) = 19.4 \ln 92 + 18 = 105.72$ millimeters of mercury.

46. a. $5 = \log \frac{I}{I_0}$, $\frac{I}{I_0} = 10^5$, or $I = 10^5 I_0 = 100{,}000\ I_0$.

b. $8 = \log \frac{I}{I_0}$, from which we find, $I = 10^8 I_0$ and so it is 1000 times greater.

c. $8.2 = \log \frac{I}{I_0}$ gives $I = 10^{8.2} I_0$. So it is $\frac{10^{8.2}}{10^5} = 10^{3.2}$, or 1585 times greater than one with magnitude 5.

47. a. $30 = 10 \log \frac{I}{I_0}$; $3 = \log \frac{I}{I_0}$; $\frac{I}{I_0} = 10^3 = 1000$. So $I = 1000\ I_0$.

b. When $D = 80$, $I = 10^8 I_0$ and when $D = 30$, $I = 10^3 I_0$. Therefore, an 80–decibel sound is $10^8/10^3$ or $10^5 = 100{,}000$ times louder than a 30–decibel sound.

c. It is $10^{15}/10^8 = 10^7$, or 10,000,000, times louder.

48. We solve the equation $29.92e^{-0.2x} = 20$, $e^{-0.2x} = \frac{20}{29.92} = 0.6684$; $-0.2x = \ln 0.6684$,

and $x = -\dfrac{\ln 0.6684}{0.2} \approx 2.01$. So, the balloonist's altitude is 2.01 miles.

49. We solve the following equation for t. Thus,

$$\frac{160}{1+240e^{-0.2t}} = 80; \quad 1+240e^{-0.2t} = \frac{160}{80},$$

$$240e^{-0.2t} = 2-1=1; \; e^{-0.2t} = \frac{1}{240}; \; -0.2t = \ln\frac{1}{240}$$

$$t = -\frac{1}{0.2}\ln\frac{1}{240} \approx 27.40$$, or approximately 27.4 years old.

50. a. The temperature when it was first poured is given by

$$T(0) = 70+100e^{0} = 170, \text{ or } 170\ {}^{\circ}F$$

b. We solve the equation

$$70+100e^{-0.0446t} = 120; \quad 100e^{-0.0446t} = 50; \quad e^{-0.0446t} = \frac{50}{100} = \frac{1}{2}$$

$$\ln e^{-0.0446t} = \ln\frac{1}{2} = \ln 1 - \ln 2 = -\ln 2; \; -0.0446t = -\ln 2$$

$$t = \frac{\ln 2}{0.0446} \approx 15.54.$$

So, it will take approximately 15.54 minutes.

51. We solve the following equation for t:

$$200(1-0.956e^{-0.18t}) = 140$$

$$1-0.956e^{-0.18t} = \frac{140}{200} = 0.7$$

$$-0.956e^{-0.18t} = 0.7-1 = -0.3$$

$$e^{-0.18t} = \frac{0.3}{0.956}$$

$$-0.18t = \ln\left(\frac{0.3}{0.956}\right)$$

$$t = -\frac{\ln\left(\frac{0.3}{0.956}\right)}{0.18} \approx 6.43875.$$

So, its approximate age is 6.44 years.

52. a. We solve the equation $0.08(1-e^{-0.02t}) = 0.02$, obtaining

$$1-e^{-0.02t}=\frac{0.02}{0.08}=\frac{1}{4};\ -e^{-0.02t}=\frac{1}{4}-1=-\frac{3}{4};\ e^{-0.02t}=\frac{3}{4}$$

$$\ln e^{-0.02t}=\ln\frac{3}{4};\ -0.02t=\ln\frac{3}{4};\ t\approx 14.38,\quad \text{or 14.38 seconds.}$$

b. $1-e^{-0.02t}=\frac{0.04}{0.08};\ -e^{-0.02t}=\frac{1}{2}-1=-\frac{1}{2};\ t\approx 34.66,\quad$ or 34.66 seconds.

53. a. We solve the equation $0.08+0.12e^{-0.02t}=0.18$.

$$0.12e^{-0.02t}=0.1;\ e^{-0.02t}=\frac{0.1}{0.12}=\frac{1}{1.2}$$

$$\ln e^{-0.02t}=\ln\frac{1}{1.2}=\ln 1-\ln 1.2=-\ln 1.2$$

$$-0.02t=-\ln 1.2$$

$$t=\frac{\ln 1.2}{0.02}\approx 9.116,\quad \text{or 9.12 sec.}$$

b. We solve the equation $0.08+0.12e^{-0.02t}=0.16$.

$$0.12e^{-0.02t}=0.08;\ e^{-0.02t}=\frac{0.08}{0.12}=\frac{2}{3};\ -0.02t=\ln\frac{2}{3}$$

$$t=-\frac{\ln\left(\frac{2}{3}\right)}{0.02}\approx 20.2733,\quad \text{or 20.27 sec.}$$

EXERCISES 5.3, page 359

1. $A=2500\left(1+\frac{0.07}{2}\right)^{20}=4974.47,\quad$ or \$4974.47.

2. $A=12{,}000\left(1+\frac{0.08}{4}\right)^{40}=26{,}496.48,\quad$ or \$26,496.48.

3. $A=150{,}000\left(1+\frac{0.1}{12}\right)^{48}=223{,}403.11,\quad$ or \$223,403.11

4. $A=150{,}000\left(1+\frac{0.09}{365}\right)^{1095}=196{,}488.13\quad$ or \$196,488.13.

5. a. Using the formula $r_{eff} = \left(1+\frac{r}{m}\right)^m - 1$ with $r = 0.10$ and $m = 2$, we have

$$r_{eff} = \left(1+\frac{0.10}{2}\right)^2 - 1 = 0.1025, \quad \text{or } 10.25 \text{ percent/yr}$$

b. Using the formula $r_{eff} = \left(1+\frac{r}{m}\right)^m - 1$ with $r = 0.09$ and $m = 4$, we have

$$r_{eff} = \left(1+\frac{0.09}{4}\right)^4 - 1 = 0.09308, \text{ or } 9.308 \text{ percent/yr.}$$

6. a. Using the formula $r_{eff} = \left(1+\frac{r}{m}\right)^m - 1$ with $r = 0.08$ and $m = 12$, we have

$$r_{eff} = \left(1+\frac{0.08}{12}\right)^{12} - 1 = 0.08300, \quad \text{or } 8.3 \text{ percent/yr.}$$

b. The effective rate is given by $R = \left(1+\frac{0.08}{365}\right)^{365} - 1 = 0.08328,$ or 8.328 percent/yr.

7. a. The present value is given by $P = 40{,}000\left(1+\frac{0.08}{2}\right)^{-8} = 29{,}227.61,$ or \$29,227.61.

b. The present value is given by $P = 40{,}000\left(1+\frac{0.08}{4}\right)^{-16} = 29{,}137.83,$ or \$29,137.83.

8. a. The present value is given by

$$P = 40{,}000\left(1+\frac{0.07}{12}\right)^{-48} = 30{,}255.95, \quad \text{or } \$30{,}255.95.$$

b. The present value is given by

$$P = 40{,}000\left(1+\frac{0.09}{365}\right)^{-(365)(4)} = 27{,}908.29, \quad \text{or } \$27{,}908.29.$$

9. $A = 5000e^{0.08(4)} \approx 6885.64,$ or \$6,885.64.

10. $A = 25000(1+0.07)^6 \approx 37{,}518.26,$ or approximately \$37,518.26.

The interest earned is \$12,518.26.

11. We use formula (6) with $A = 7500$, $P = 5000$, $m = 12$, and $t = 3$. Thus

$$7500 = 5000\left(1+\tfrac{r}{12}\right)^{36};$$

$$\left(1+\tfrac{r}{12}\right)^{36} = \tfrac{7500}{5000} = \tfrac{3}{2},\ \ln(1+\tfrac{r}{12})^{36} = \ln 1.5;$$

$$36\left(1+\tfrac{r}{12}\right) = \ln 1.5$$

$$\left(1+\tfrac{r}{12}\right) = \tfrac{\ln 1.5}{36} = 0.0112629$$

$$1+\tfrac{r}{12} = e^{0.0112629} = 1.011327; \tfrac{r}{12} = 0.011327;$$

$$r = 0.13592$$

So the interest rate is 13.59% per year.

12. We use formula (6) with $A = 7500$, $P = 5000$, $m = 4$, and $t = 3$. Thus

$$7500 = 5000\left(1+\tfrac{r}{4}\right)^{12}$$

$$\left(1+\tfrac{r}{4}\right)^{12} = \tfrac{7500}{5000} = \tfrac{3}{2},\ \ln(1+\tfrac{r}{4})^{12} = \ln 1.5;$$

$$12\left(1+\tfrac{r}{4}\right) = \ln 1.5; \quad \left(1+\tfrac{r}{4}\right) = \tfrac{\ln 1.5}{12} = 0.0337888$$

$$1+\tfrac{r}{4} = e^{0.0337888} = 1.034366;\ \tfrac{r}{4} = 0.034366$$

$$r = 0.034366$$

So the required interest rate is 13.75% per year.

13. We use formula (6) with $A = 8000$, $P = 4000$, $m = 2$, and $t = 4$. Thus

$$8000 = 4000\left(1+\tfrac{r}{2}\right)^{8}$$

$$\left(1+\tfrac{r}{2}\right)^{8} = \tfrac{8000}{5000} = 1.6,\ \ln(1+\tfrac{r}{2})^{8} = \ln 1.6;$$

$$8\ln\left(1+\tfrac{r}{2}\right) = \ln 1.6$$

$$\ln\left(1+\tfrac{r}{2}\right) = \tfrac{\ln 1.6}{8} = 0.05875$$

$$1+\tfrac{r}{2} = e^{0.05875} = 1.06051;\ \tfrac{r}{2} = 0.06051$$

$$r = 0.1210$$

So the required interest rate is 12.1% per year.

14. We use formula (6) with $A = 5500$, $P = 5000$, $m = 12$, and $t = \tfrac{1}{2}$. Thus

$$5500 = 5000\left(1+\tfrac{r}{12}\right)^{6}; \quad \left(1+\tfrac{r}{12}\right)^{6} = \tfrac{5500}{5000} = 1.1$$

Proceeding as in the previous problem, we find $r = 0.1921$. So the required interest rate is 19.21% per year.

15. We use formula (6) with $A = 4000$, $P = 2000$, $m = 1$, and $t = 5$. Thus

$$4000 = 2000(1+r)^5; \quad (1+r)^5 = 2\,; 5\ln(1+r) = \ln 2; \;\ \ln(1+r) = \tfrac{\ln 2}{5} = 0.138629$$

$$1+r = e^{0.138629} = 1.148698; \; r = 0.1487$$

So the required interst rate is 14.87% per year.

16. We use formula (6) with $A = 6000$, $P = 2000$, $m = 12$, and $t = 5$. Thus

$$6000 = 2000\left(1+\frac{r}{12}\right)^{60}$$

Proceeding as in the previous problem, we find $r = 22.17$. So the required interest rate is 22.17% per year.

17. We use formula (6) with $A = 6500$, $P = 5000$, $m = 12$, and $r = 0.12$. Thus

$$6500 = 5000\left(1+\frac{0.12}{12}\right)^{12t}; \quad (1.01)^{12t} = \frac{6500}{5000} = 1.3; \quad 12t\ln(1.01) = \ln 1.3$$

$$t = \frac{\ln 1.3}{12\ln 1.01} \approx 2.197$$

So, it will take approximately 2.2 years.

18. We use formula (6) with $A = 15000$, $P = 12000$, $m = 12$, and $r = 0.08$. Thus,

$$15000 = 12000\left(1+\tfrac{0.08}{12}\right)^{12t}$$

Proceeding as in the previous exercise, we find $r = 2.799$. So it will take approximately 2.8 years.

19. We use formula (6) with $A = 4000$, $P = 2000$, $m = 12$, and $r = 0.09$. Thus,

$$4000 = 2000\left(1+\tfrac{0.09}{12}\right)^{12t}$$

$$\left(1+\tfrac{0.09}{12}\right)^{12t} = 2$$

$$12\text{t} \ln\left(1+\tfrac{0.09}{12}\right) = \ln 2 \;\text{ and }\; t = \frac{\ln 2}{12\ln\left(1+\frac{0.09}{12}\right)} \approx 7.73.$$

So it will take approximately 7.7 years.

20. We use formula (6) with $A = 15000$, $P = 5000$, $m = 365$, and $r = 0.08$. Thus

$$15000 = 5000\left(1+\tfrac{0.08}{365}\right)^{365t}$$

to obtain $t = \dfrac{\ln\left(\frac{15000}{5000}\right)}{365\ln\left(1+\frac{0.08}{365}\right)} \approx 13.73$. So, it will take approximately 13.7 years.

21. We use formula (10) with $A = 6000$, $P = 5000$, and $t = 3$. Thus,

$$6000 = 5000e^{3r}$$

$$e^{3r} = \frac{6000}{5000} = 1.2; \qquad 3r = \ln 1.2$$

$$r = \frac{\ln 1.2}{3} \approx 0.6077$$

So the interest rate is 6.08% per year.

22. We use formula (10) with $A = 8000$, $P = 4000$, and $t = 5$. Thus

$$8000 = 4000e^{5r}$$

obtaining $r = \dfrac{\ln\left(\frac{8000}{4000}\right)}{5} \approx 0.13863$. So the interest rate is 13.86% per year.

23. We use formula (10) formula (6) with $A = 7000$, $P = 6000$, and $r = 0.075$. Thus

$$7000 = 6000e^{0.075t}; \quad e^{0.075t} = \tfrac{7000}{6000} = \tfrac{7}{6}$$

$$0.075t \ln e = \ln\tfrac{7}{6} \quad \text{and} \quad t = \frac{\ln\frac{7}{6}}{0.075} \approx 2.055.$$

So, it will take 2.06 years.

24. We use formula (10) with $A = 16{,}000$, $P = 8000$, and $r = 0.08$. Thus

$$16{,}000 = 8000e^{0.08t}$$

obtaining $t = \dfrac{\ln 2}{0.08} \approx 8.664$. So, it will take 8.7 years.

25. The Estradas can expect to pay $80{,}000(1+0.09)^4$, or approximately \$112,926.52.

26. The utility company will have to increase its generating capacity by a factor of $(1.08)^{10}$, or 2.16 times.

27. The investment will be worth

$A = 1.5\left(1+\dfrac{0.095}{2}\right)^{20} = 3.794651$, or approximately \$3.8 million dollars.

28. Bernie originally invested $P = 22{,}289.22\left(1+\dfrac{0.08}{4}\right)^{-20} = 15{,}000$, or \$15,000.

29. The present value of the \$8000 loan due in 3 years is given by

$$P = 8000\left(1+\frac{0.10}{2}\right)^{-6} = 5969.72, \text{ or } \$5969.72.$$

The present value of the \$15,000 loan due in 6 years is given by

$$P = 15{,}000\left(1+\frac{0.10}{2}\right)^{-12} = 8352.56, \text{ or } \$8352.56.$$

Therefore, the amount the proprietors of the inn will be required to pay at the end of 5 years is given by $A = 14{,}322.28\left(1+\dfrac{0.10}{2}\right)^{10} = 23{,}329.48$, or \$23,329.48.

30. a. The accumulated amount before taxes is $A = 25{,}000\left(1+\dfrac{0.12}{1}\right)^{10} \approx 77{,}646.21$.

After taxes, it is worth \$55,905.27.

b. The accumulated amount (tax-free) is $A = 25{,}000(1+0.864)^{10} \approx 57{,}258.19$, or \$57,258.19.

31. We solve the equation $2 = 1(1+0.075)^t$ for t. Taking the logarithm on both sides, we have $\ln 2 = \ln(1.075)^t \approx t\ln 1.075$. So $t = \dfrac{\ln 2}{\ln 1.075} \approx 9.58$, or 9.6 years.

32. We solve the equation $216{,}000 160{,}000(1+R)^6$, $(1+R)^6 = 1.35$,
$1+R = (1.35)^{1/6} \approx 1.051289$ and $r = 0.051289$, or 5.13%.

33. The effective annual rate of return on his investment is found by solving the equation $(1+r)^2 = \dfrac{32100}{25250}$

$$1+r = \left(\frac{32100}{25250}\right)^{1/2}$$

$1+r \approx 1.1275$ and $r = 0.1275$, or 12.75 percent.

34. Suppose \$1 is invested in each investment.

Investment A: Accumulated amount is $\left(1+\frac{0.1}{2}\right)^8 \approx 1.47746$.

Investment B: Accumulated amount is $e^{0.0975(4)} \approx 1.47698$.
So Investment A has a higher rate of return.

35. $P = Ae^{-rt} = 59673e^{-(0.08)5} \approx 40{,}000.008$, or approximately \$40,000.

36. We solve the equation $3.6 = 1.4e^{6r}$, $e^{6r} = \dfrac{3.6}{1.4}$, or $6r \ln e \approx \ln \dfrac{3.6}{1.4}$,
$6r = 0.944462$, $r = 0.1574$, or approximately 15.7%.

37. a. If they invest the money at 10.5 percent compounded quarterly, they should set aside $P = 70{,}000\left(1 + \frac{0.105}{4}\right)^{-28} \approx 33{,}885.14$, or \$33,885.14.

b. If they invest the money at 10.5 percent compounded continuously, they should set aside $P = 70{,}000e^{-0.735} = 33{,}565.38$, or \$33,565.38.

EXERCISES 5.4 , page 368

1. $f(x) = e^{3x}$; $f'(x) = 3e^{3x}$

2. $f(x) = 3e^{x}$, $f'(x) = 3e^{x}$.

3. $g(t) = e^{-t}$; $g'(t) = -e^{-t}$

4. $f(x) = e^{-2x}$; $f'(x) = -2e^{-2x}$

5. $f(x) = e^{x} + x$; $f'(x) = e^{x} + 1$

6. $f(x) = 2e^{x} - x^{2}$, $f'(x) = 2e^{x} - 2x = 2(e^{x} - x)$.

7. $f(x) = x^{3}e^{x}$, $f'(x) = x^{3}e^{x} + e^{x}(3x^{2}) = x^{2}e^{x}(x + 3)$.

8. $f(u) = u^{2}e^{-u}$, $f'(u) = 2ue^{-u} + u^{2}e^{-u}(-1) = u(2 - u)e^{-u}$.

9. $f(x)=\dfrac{2e^x}{x}$, $f'(x)=\dfrac{x(2e^x)-2e^x(1)}{x^2}=\dfrac{2e^x(x-1)}{x^2}$.

10. $f(x)=\dfrac{x}{e^x}$; $f'(x)=\dfrac{e^x(1)-xe^x}{e^{2x}}=\dfrac{1-x}{e^x}$.

11. $f(x)=3(e^x+e^{-x})$; $f'(x)=3(e^x-e^{-x})$. 12. $f(x)=\dfrac{e^x+e^{-x}}{2}$; $f'(x)=\dfrac{e^x-e^{-x}}{2}$.

13. $f(w)=\dfrac{e^w+1}{e^w}=1+\dfrac{1}{e^w}=1+e^{-w}$. $f'(w)=-e^{-w}=-\dfrac{1}{e^w}$.

14. $f(x)=\dfrac{e^x}{e^x+1}$; $f'(x)=\dfrac{(e^x+1)e^x-e^x(e^x)}{(e^x+1)^2}=\dfrac{e^x}{(e^x+1)^2}$.

15. $f(x)=2e^{3x-1}$, $f'(x)=2e^{3x-1}(3)=6e^{3x-1}$.

16. $f(t)=4e^{3t+2}$; $f'(t)=4e^{3t+2}(3)=12e^{3t+2}$ 17. $h(x)=e^{-x^2}$; $h'(x)=e^{-x^2}(-2x)=-2xe^{-x^2}$.

18. $f(x)=e^{x^2-1}$; $f'(x)=e^{x^2-1}(2x)=2xe^{x^2-1}$.

19. $f(x)=3e^{-1/x}$; $f'(x)=3e^{-1/x}\cdot\dfrac{d}{dx}\left(-\dfrac{1}{x}\right)=3e^{-1/x}\left(\dfrac{1}{x^2}\right)=\dfrac{3e^{-1/x}}{x^2}$.

20. $f(x)=e^{1/(2x)}$; $f'(x)=e^{1/2x}\cdot\dfrac{d}{dx}\left(\dfrac{1}{2x}\right)=\dfrac{1}{2}e^{1/2x}\cdot -x^{-2}=-\dfrac{e^{1/2x}}{2x^2}$.

21. $f(x)=(e^x+1)^{25}$, $f'(x)=25(e^x+1)^{24}e^x=25e^x(e^x+1)^{24}$.

22. $f(x)=(4-e^{-3x})^3$; $f'(x)=3(4-e^{-3x})^2(-e^{-3x})(-3)=9e^{-3x}(4-e^{-3x})^2$.

23. $f(x)=e^{\sqrt{x}}$; $f'(x)=e^{\sqrt{x}}\dfrac{d}{dx}x^{1/2}=e^{\sqrt{x}}\dfrac{1}{2}x^{-1/2}=\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$.

24. $f(t)=-e^{-\sqrt{2t}}$; $f'(t)=-e^{-\sqrt{2t}}\dfrac{d}{dt}(-\sqrt{2t})=e^{-\sqrt{2t}}\left(\dfrac{1}{2}\right)(2t)^{-1/2}(2)=\dfrac{e^{-\sqrt{2t}}}{\sqrt{2t}}$.

25. $f(x)=(x-1)e^{3x+2}$; $f'(x)=(x-1)(3)e^{3x+2}+e^{3x+2}=e^{3x+2}(3x-3+1)=e^{3x+2}(3x-2)$.

26. $f(s)=(s^2+1)e^{-s^2}$; $f'(s)=2se^{-s^2}+(s^2+1)e^{-s^2}(-2s)=-2s^3e^{-s^2}$.

27. $f(x)=\dfrac{e^x-1}{e^x+1}$; $f'(x)=\dfrac{(e^x+1)(e^x)-(e^x-1)(e^x)}{(e^x+1)^2}=\dfrac{e^x(e^x+1-e^x+1)}{(e^x+1)^2}=\dfrac{2e^x}{(e^x+1)^2}$.

28. $g(t)=\dfrac{e^{-t}}{1+t^2}$;

$$g'(t)=\frac{(1+t^2)(-e^{-t})-(e^{-t})(2t)}{(1+t^2)^2}=\frac{e^{-t}(-1-t^2-2t)}{(1+t^2)^2}=\frac{-e^{-t}(t^2+2t+1)}{(1+t^2)^2}=\frac{-e^{-t}(t+1)^2}{(1+t^2)^2}.$$

29. $f(x)=e^{-4x}+2e^{3x}; f'(x)=-4e^{-4x}+6e^{3x}$ and
$f''(x)=16e^{-4x}+18e^{3x}=2(8e^{-4x}+9e^{3x})$.

30. $f(t)=3e^{-2t}-5e^{-t}$; $f'(t)=-6e^{-2t}+5e^{-t}$ and $f''(t)=12e^{-2t}-5e^{-t}$.

31. $f(x)=2xe^{3x}; f'(x)=2e^{3x}+2xe^{3x}(3)=2(3x+1)e^{3x}$.
$f''(x) \quad =6e^{3x}+2(3x+1)e^{3x}(3)=6(3x+2)e^{3x}$.

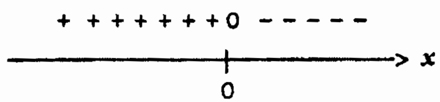
32. $f(t)=t^2e^{-2t}$; $f'(t)=2te^{-2t}+t^2e^{-2t}(-2)=2t(1-t)e^{-2t}$.
$f''(t)=(2-4t)e^{-2t}+2t(1-t)e^{-2t}(-2)=2(2t^2-4t+1)e^{-2t}$.

33. $y=f(x)=e^{2x-3}$. $f'(x)=2e^{2x-3}$. To find the slope of the tangent line to the graph of f at $x=3/2$, we compute $f'(\frac{3}{2})=2e^{3-3}=2$. Next, using the point–slope form of the equation of a line, we find that

$$y-1=2(x-\tfrac{3}{2})$$
$$=2x-3, \qquad \text{or} \qquad y=2x-2.$$

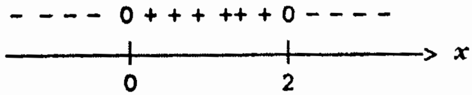
34. $y=e^{-x^2}$. The slope of the tangent line at any point is $y'=e^{-x^2}(-2x)=-2xe^{-x^2}$. The slope of the tangent line when $x=1$ is $m=-2e^{-1}$. Therefore, an equation of the tangent line is $y-\frac{1}{e}=-\frac{2}{e}(x-1)$, or $y=-\frac{2}{e}+\frac{3}{e}$.

35. $f(x)=e^{-x^2/2}$, $f'(x)=e^{-x^2/2}(-x)=-xe^{-x^2/2}$. Setting $f'(x)=0$, gives $x=0$ as the only critical point of f. From the sign diagram,



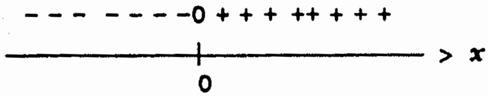
we conclude that f is increasing on $(-\infty,0)$ and decreasing on $(0,\infty)$.

36. $f(x) = x^2e^{-x}$; $f'(x) = 2xe^{-x} + x^2e^{-x}(-1) = x(2-x)e^{-x}$. Observe that $f'(x) = 0$ if $x = 0$ or 2. The sign diagram of f'



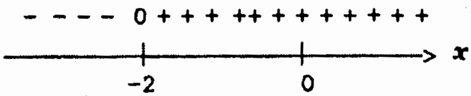
shows that f is increasing on (0,2) and decreasing on $(-\infty,0) \cup (2,\infty)$.

37. $f(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$, $f'(x) = \frac{1}{2}(e^x + e^{-x})$, $f''(x) = \frac{1}{2}(e^x - e^{-x})$. Setting $f''(x) = 0$, gives $e^x = e^{-x}$ or $e^{2x} = 1$, and $x = 0$. From the sign diagram for f'',



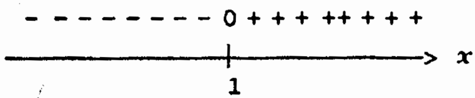
we conclude that f is concave upward on $(0,\infty)$ and concave downward on $(-\infty,0)$.

38. $f(x) = xe^x$. $f'(x) = e^x + xe^x = (x+1)e^x$. $f''(x) = (x+1)e^x + e^x = (x+2)e^x$. Setting $f''(x) = 0$ gives $x = -2$. The sign diagram of f''



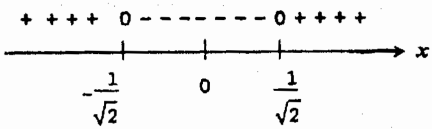
shows that f is concave downward on $(-\infty,-2)$ and concave upward on $(-2,\infty)$.

39. $f(x) = xe^{-2x}$. $f'(x) = e^{-2x} + xe^{-2x}(-2) = (1-2x)e^{-2x}$.
$f''(x) = -2e^{-2x} + (1-2x)e^{-2x}(-2) = 4(x-1)e^{-2x}$.
Observe that $f''(x) = 0$ if $x = 1$. The sign diagram of f''



shows that $(1, e^{-2})$ is an inflection point.

40. $f(x) = 2e^{-x^2} = 2(e^{-x^2})$, $f'(x) = 2(-2x)e^{-x^2} = -4xe^{-x^2}$.
$f''(x) = -4x(-2x)e^{-x^2} - 4e^{-x^2} = -4e^{-x^2}(-2x^2+1) = 4e^{-x^2}(2x^2-1)$.
Setting $f''(x) = 0$ gives $2x^2 = 1$, $x^2 = 1/2$, or $x = \pm\frac{\sqrt{2}}{2}$.
The sign diagram for f'' is



We see that $(-\frac{\sqrt{2}}{2}, 2e^{-1/2})$ and $(\frac{\sqrt{2}}{2}, 2e^{-1/2})$ are inflection points.

41. $f(x) = e^{-x^2}$. $f'(x) = -2xe^{-x^2} = 0$ if $x = 0$, the only critical point of f.

x	-1	0	1
$f(x)$	e^{-1}	1	e^{-1}

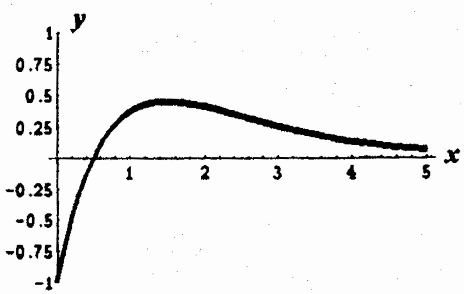
From the table, we see that f has an absolute minimum value of e^{-1} attained at $x = -1$ and $x = 1$. It has an absolute maximum at (0,1).

42. $h(x) = e^{x^2-4}$, $h'(x) = 2xe^{x^2-4}$. Setting $h'(x) = 0$ gives $x = 0$ as the only critical point of h.

x	-2	0	2
$h(x)$	1	e^{-4}	1

We see that $h(0) = e^{-4}$ is the absolute minimum and $h(-2) = 1$ and $h(2) = 1$ are the absolute maximum values of h.

43. $g(x) = (2x - 1)e^{-x}$; $g'(x) = 2e^{-x} + (2x - 1)e^{-x}(-1) = (3 - 2x)e^{-x} = 0$, if $x = 3/2$. The graph of g shows that $(\frac{3}{2}, 2e^{-3/2})$ is an absolute maximum, and (0,–1) is an absolute minimum.

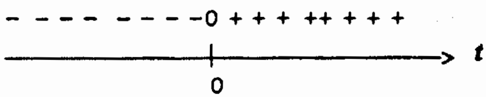


44. $f(x) = xe^{-x^2}$; $f'(x) = e^{-x^2} + xe^{-x^2}(-2x) = (1 - 2x^2)e^{-x^2} = 0$, if $x = \pm\frac{\sqrt{2}}{2}$.

x	0	$\frac{\sqrt{2}}{2}$	2
$f(x)$	0	$\frac{\sqrt{2}}{2}e^{-1/2}$	$2e^{-4}$

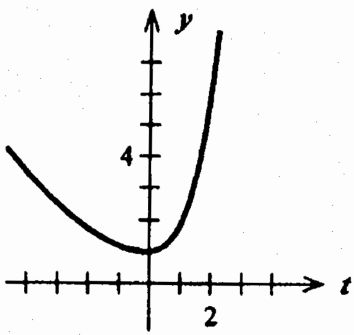
From the table, we see that f has an absolute minimum at (0,0) and an absolute maximum at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}e^{-1/2})$.

45. $f(t) = e^t - t$;
We first gather the following information on f.
1. The domain of f is $(-\infty,\infty)$.
2. Setting $t = 0$ gives 1 as the y–intercept.
3. $\lim_{t\to-\infty}(e^t - t) = \infty$ and $\lim_{t\to\infty}(e^t - t) = \infty$.
4. There are no asymptotes.
5. $f'(t) = e^t - 1$ if $t = 0$, a critical point of f. From the sign diagram for f'



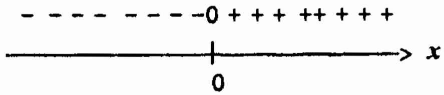
we see that f is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.
6. From the results of (5), we see that (0,1) is a relative minimum of f.
7. $f''(t) = e^t > 0$ for all t in $(-\infty,\infty)$. So the graph of f is concave upward on $(-\infty,\infty)$.
8. There are no inflection points.

The graph of f follows.



46. $h(x) = \dfrac{e^x + e^{-x}}{2}$.
We first gather the following information on h.

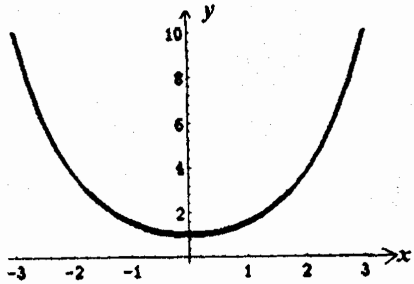
1. The domain of h is $(-\infty,\infty)$.
2. Setting $x = 0$ gives 1 as the y–intercept.
3. $\lim_{x\to-\infty} h(x) = \lim_{x\to\infty} h(x) = \infty$.
4. There are no asymptotes.
5. $h'(x) = \frac{1}{2}(e^x - e^{-x}) = 0$ if $e^x = e^{-x}$, $e^{2x} = 1$ or $x = 0$, a critical point of h. The sign diagram of h'



shows that h is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.

6. The results of (5) show that (0,1) is a relative minimum of h.
7. $h''(x) = \frac{1}{2}(e^x + e^{-x})$ is always positive. So the graph of h is always concave upward.
8. The results of (7) show that h has no inflection points.

The graph of h follows.

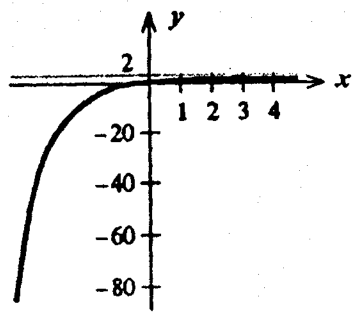


47. $f(x) = 2 - e^{-x}$.

We first gather the following information on f.

1. The domain of f is $(-\infty,\infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.
3. $\lim_{x\to-\infty} (2 - e^{-x}) = -\infty$ and $\lim_{x\to\infty} (2 - e^{-x}) = 2$,
4. From the results of (3), we see that $y = 2$ is a horizontal asymptote of f.
5. $f'(x) = e^{-x}$. Observe that $f'(x) > 0$ for all x in $(-\infty,\infty)$ and so f is increasing on $(-\infty,\infty)$.
6. Since there are no critical points, f has no relative extrema.
7. $f''(x) = -e^{-x} < 0$ for all x in $(-\infty,\infty)$ and so the graph of f is concave downward on $(-\infty,\infty)$.

8. There are no inflection points
The graph of f follows.



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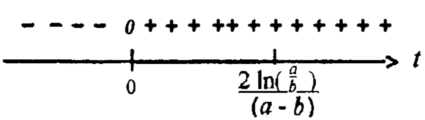
70. a. $y' = c(-be^{-bt} + ae^{-at}) = ca(-\frac{b}{a}e^{-bt} + e^{-at}) = cae^{-at}(-\frac{b}{a}e^{(a-b)t} + 1)$.

Setting $y' = 0$ gives $-\dfrac{b}{a}e^{(a-b)t} + 1 = 0$; $e^{(a-b)t} = \dfrac{a}{b}$; $\ln e^{(a-b)t} = \ln\left(\frac{a}{b}\right)$; $t = \dfrac{\ln\left(\frac{a}{b}\right)}{a-b}$.

Since $y(0) = 0$ and $\lim\limits_{t\to\infty} y = 0$, $t = \dfrac{\ln\left(\frac{a}{b}\right)}{a-b}$ gives the time at which the concentration is maximal.

b. $y'' = c(b^2e^{-bt} - a^2e^{-at}) = ca^2e^{-at}\left(\frac{b^2}{a^2}e^{(a-b)t} - 1\right)$. Setting $y'' = 0$ gives $e^{(a-b)t} = \dfrac{a^2}{b^2}$; $t = \dfrac{2\ln\left(\frac{a}{b}\right)}{(a-b)}$. From the sign diagram of y''



we see that the concentration of the drug is increasing most rapidly when $t = \dfrac{2\ln\left(\frac{a}{b}\right)}{(a-b)}$.

EXERCISES 5.5, page 379

1. $f(x) = 5\ \ln x; f'(x) = 5\left(\frac{1}{x}\right) = \frac{5}{x}.$

2. $f(x) = \ln 5x;\ f'(x) = \frac{5}{5x} = \frac{1}{x}.$

3. $f(x) = \ln (x + 1); f'(x) = \frac{1}{x+1}.$

4. $g(x) = \ln (2x + 1); g'(x) = \frac{2}{2x+1}.$

5. $f(x) = \ln x^8; f'(x) = \frac{8x^7}{x^8} = \frac{8}{x}.$

6. $h(t) = 2\ \ln t^5;\ h(t) = 10\ \ln t$ and so $h'(t) = \frac{10}{t}.$

7. $f(x) = \ln x^{1/2};\quad f'(x) = \frac{\frac{1}{2}x^{-1/2}}{x^{1/2}} = \frac{1}{2x}.$

8. $f(x) = \ln (x^{1/2} + 1);\quad f'(x) = \frac{\frac{1}{2}x^{-1/2}}{x^{1/2}+1} = \frac{1}{2\sqrt{x}(\sqrt{x}+1)}.$

9. $f(x) = \ln\left(\frac{1}{x^2}\right) = \ln x^{-2} = -2\ \ln x;\quad f'(x) = -\frac{2}{x}.$

10. $f(x) = \ln\ \frac{1}{2x^3} = \ln 1 - \ln (2x^3) = -\ln 2 - 3\ \ln x.$ So $f'(x) = -\frac{3}{x}.$

11. $f(x) = \ln (4x^2 - 6x + 3);\quad f'(x) = \frac{8x-6}{4x^2-6x+3} = \frac{2(4x-3)}{4x^2-6x+3}.$

12. $f(x) = \ln (3x^2 - 2x + 1);\quad f'(x) = \frac{6x-2}{3x^2-2x+1} = \frac{2(3x-1)}{3x^2-2x+1}.$

13. $f(x) = \ln\left(\frac{2x}{x+1}\right) = \ln 2x - \ln (x + 1).$

$$f'(x) = \frac{2}{2x} - \frac{1}{x+1} = \frac{2(x+1)-2x}{2x(x+1)} = \frac{2x+2-2x}{2x(x+1)} = \frac{2}{2x(x+1)} = \frac{1}{x(x+1)}.$$

14. $f(x) = \ln (x + 1) - \ln (x - 1).$

So $f'(x) = \dfrac{1}{x+1} - \dfrac{1}{x-1} = \dfrac{(x-1)-(x+1)}{x^2-1} = -\dfrac{2}{x^2-1}$.

15. $f(x) = x^2 \ln x$; $f'(x) = x^2\left(\frac{1}{x}\right) + (\ln x)(2x) = x + 2x \ln x = x(1 + 2 \ln x)$

16. $f(x) = 3x^2 \ln 2x$; $f'(x) = 6x \ln 2x + 3x^2 \cdot \dfrac{2}{2x} = 6x \ln 2x + 3x = 3x(2 \ln 2x + 1)$.

17. $f(x) = \dfrac{2 \ln x}{x}$. $f'(x) = \dfrac{x\left(\frac{2}{x}\right) - 2 \ln x}{x^2} = \dfrac{2(1-\ln x)}{x^2}$.

18. $f(x) = \dfrac{3 \ln x}{x^2}$. $f'(x) = \dfrac{x^2\left(\frac{3}{x}\right) - (3 \ln x)(2x)}{x^4} = \dfrac{3x(1-2\ln x)}{x^4}$.

19. $f(u) = \ln (u-2)^3$; $f'(u) = \dfrac{3(u-2)^2}{(u-2)^3} = \dfrac{3}{u-2}$.

20. $f(x) = \ln (x^3-3)^4 = 4 \ln (x^3-3)$. $f'(x) = \dfrac{4(3x^2)}{x^3-3} = \dfrac{12x^2}{x^3-3}$.

21. $f(x) = (\ln x)^{1/2}$ and $f'(x) = \dfrac{1}{2}(\ln x)^{-1/2}\left(\dfrac{1}{x}\right) = \dfrac{1}{2x\sqrt{\ln x}}$.

22. $f(x) = (\ln x + x)^{1/2}$; $f'(x) = \dfrac{1}{2}(\ln x + x)^{-1/2}\left(\dfrac{1}{x}+1\right) = \dfrac{x+1}{2x\sqrt{\ln x + x}}$.

23. $f(x) = (\ln x)^3$; $f'(x) = 3(\ln x)^2\left(\dfrac{1}{x}\right) = \dfrac{3(\ln x)^2}{x}$.

24. $f(x) = 2(\ln x)^{3/2}$; $f'(x) = 2\left(\dfrac{3}{2}\right)(\ln x)^{1/2}\left(\dfrac{1}{x}\right) = \dfrac{3(\ln x)^{1/2}}{x}$.

25. $f(x) = \ln (x^3+1)$; $f'(x) = \dfrac{3x^2}{x^3+1}$.

26. $f(x) = \ln (x^2-4)^{1/2} = \frac{1}{2} \ln (x^2-4)$. So $f'(x) = \dfrac{2x}{2(x^2-4)} = \dfrac{x}{x^2-4}$.

27. $f(x) = e^x \ln x$. $f'(x) = e^x \ln x + e^x\left(\dfrac{1}{x}\right) = \dfrac{e^x(x \ln x + 1)}{x}$.

28. $f(x) = e^x \ln\sqrt{x+3} = \frac{1}{2} e^x \ln(x+3)$.

$$f'(x) = \frac{1}{2}\left[e^x \ln(x+3) + e^x \cdot \frac{1}{x+3}\right] = \frac{e^x[(x+3)\ln(x+3)+1]}{2(x+3)}.$$

29. $f(t) = e^{2t}\ln(t+1)$

$$f'(t) = e^{2t}\left(\frac{1}{t+1}\right) + \ln(t+1)\cdot(2e^{2t}) = \frac{[2(t+1)\ln(t+1)+1]e^{2t}}{t+1}.$$

30. $g(t) = t^2\ln(e^{2t}+1)$;

$$g'(t) = 2t\ln(e^{2t}+1) + t^2\left(\frac{2e^{2t}}{e^{2t}+1}\right) = \frac{2t[(e^{2t}+1)\ln(e^{2t}+1)+te^{2t}]}{e^{2t}+1}.$$

31. $f(x)\ \dfrac{\ln x}{x}$. $f'(x) = \dfrac{x(\frac{1}{x}) - \ln x}{x^2} = \dfrac{1-\ln x}{x^2}.$

32. $g(t) = \dfrac{t}{\ln t}$; $g'(t) = \dfrac{(\ln t)(1) - t(\frac{1}{t})}{(\ln t)^2} = \dfrac{\ln t - 1}{(\ln t)^2}.$

33. $f(x) = \ln 2 + \ln x$; So $f'(x) = \dfrac{1}{x}$ and $f''(x) = -\dfrac{1}{x^2}$.

34. $f(x) = \ln(x+5)$; $f'(x) = \dfrac{1}{x+5}$ and so

$$f''(x) = \frac{d}{dx}(x+5)^{-1} = -(x+5)^{-2} = -\frac{1}{(x+5)^2}.$$

35. $f(x) = \ln(x^2+2)$; $f'(x) = \dfrac{2x}{(x^2+2)}$ and

$$f''(x) = \frac{(x^2+2)(2) - 2x(2x)}{(x^2+2)^2} = \frac{2(2-x^2)}{(x^2+2)^2}.$$

36. $f(x) = (\ln x)^2$; $f'(x) = 2(\ln x)\left(\dfrac{1}{x}\right) = \dfrac{2\ln x}{x}$ and

$$f''(x) = \frac{x(\frac{2}{x}) - 2\ln x}{x^2} = \frac{2(1-\ln x)}{x^2}.$$

37. $y = (x+1)^2(x+2)^3$

$\ln y = \ln(x+1)^2(x+2)^3 = \ln(x+1)^2 + \ln(x+2)^3$

$= 2\ln(x+1) + 3\ln(x+2)$.

$$\frac{y'}{y} = \frac{2}{x+1} + \frac{3}{x+2} = \frac{2(x+2)+3(x+1)}{(x+1)(x+2)} = \frac{5x+7}{(x+1)(x+2)}$$

$$y' = \frac{(5x+7)(x+1)^2(x+2)^3}{(x+1)(x+2)} = (5x+7)(x+1)(x+2)^2.$$

38. $y = (3x+2)^4(5x-1)^2$; $\ln y = 4\ln(3x+2) + 2\ln(5x-1)$

$$\frac{dy}{dx}\cdot\frac{1}{y} = \frac{4(3)}{3x+2} + \frac{2(5)}{5x-1} = \frac{12(5x-1)+10(3x+2)}{(3x+2)(5x-1)}$$

$$= \frac{60x-12+30x+20}{(3x+2)(5x-1)} = \frac{2(45x+4)}{(3x+2)(5x-1)}.$$

$$\frac{dy}{dx} = \frac{2(3x+2)^4(5x-1)^2(45x+4)}{(3x+2)(5x-1)} = 2(3x+2)^3(5x-1)(45x+4).$$

39. $y = (x-1)^2(x+1)^3(x+3)^4$

$\ln y = 2\ln(x-1) + 3\ln(x+1) + 4\ln(x+3)$

$$\frac{y'}{y} = \frac{2}{x-1} + \frac{3}{x+1} + \frac{4}{x+3}$$

$$= \frac{2(x+1)(x+3)+3(x-1)(x+3)+4(x-1)(x+1)}{(x-1)(x+1)(x+3)}$$

$$= \frac{2x^2+8x+6+3x^2+6x-9+4x^2-4}{(x-1)(x+1)(x+3)} = \frac{9x^2+14x-7}{(x-1)(x+1)(x+3)}.$$

Therefore,

$$y' = \frac{9x^2+14x-7}{(x-1)(x+1)(x+3)}\cdot y$$

$$= \frac{(9x^2+14x-7)(x-1)^2(x+1)^3(x+3)^4}{(x-1)(x+1)(x+3)}$$

$$= (9x^2+14x-7)(x-1)(x+1)^2(x+3)^3.$$

40. $y = (3x+5)^{1/2}(2x-3)^4$; $\ln y = \frac{1}{2}\ln(3x+5) + 4\ln(2x-3)$

$$\frac{y'}{y} = \frac{1}{2}\left(\frac{1(3)}{3x+5}\right) + 4\left(\frac{2}{2x-3}\right) = \frac{3}{2(3x+5)} + \frac{8}{2x-3}$$

$$= \frac{3(2x-3)+16(3x+5)}{2(3x+5)(2x-3)} = \frac{54x+71}{2(3x+5)(2x-3)}.$$

Therefore,

$$y' = \left(\frac{54x+71}{2}\right)(3x+5)^{-1}(2x-3)^{-1}y$$

$$= \left(\frac{54x+71}{2}\right)(3x+5)^{-1}(2x-3)^{-1}(3x+5)^{1/2}(2x-3)^4$$

$$= \tfrac{1}{2}(2x-3)^3(54x+71)(3x+5)^{-1/2}.$$

41. $y = \dfrac{(2x^2-1)^5}{\sqrt{x+1}}.$

$$\ln y = \ln\frac{(2x^2-1)^5}{(x+1)^{1/2}} = 5\ \ln(2x^2-1) - \frac{1}{2}\ln(x+1)$$

So $\dfrac{y'}{y} = \dfrac{20x}{2x^2-1} - \dfrac{1}{2(x+1)} = \dfrac{40x(x+1)-(2x^2-1)}{2(2x^2-1)(x+1)}$

$$= \frac{38x^2+40x+1}{2(2x^2-1)(x+1)}.$$

$$y' = \frac{38x^2+40x+1}{2(2x^2-1)(x+1)}\cdot\frac{(2x^2-1)^5}{\sqrt{x+1}} = \frac{(38x^2+40x+1)(2x^2-1)^4}{2(x+1)^{3/2}}.$$

42. $y = \dfrac{\sqrt{4+3x^2}}{\sqrt[3]{x^2+1}}$; $\ln y = \dfrac{1}{2}\ \ln(4+3x^2) - \dfrac{1}{3}(x^2+1).$

$$\frac{y'}{y} = \frac{6x}{2(4+3x^2)} - \frac{2x}{3(x^2+1)} = \frac{9x(x^2+1)-2x(4+3x^2)}{3(4+3x^2)(x^2+1)}.$$

$$y' = \frac{3x^3+x}{3(4+3x^2)(x^2+1)}\cdot\frac{\sqrt{4+3x^2}}{(x^2+1)^{1/3}} = \frac{x(3x^2+1)}{3(4x^2+1)^{1/2}(x^2+1)^{4/3}}.$$

43. $y = 3^x$; $\quad \ln y = x\ \ln 3$; $\quad \dfrac{1}{y}\cdot\dfrac{dy}{dx} = \ln 3$; $\quad \dfrac{dy}{dx} = y\ \ln 3 = 3^x\ \ln 3.$

44. $y = x^{x+2}$. $\ln y = \ln x^{x+2} = (x+2)\ln x.$

So $\dfrac{y'}{y} = \ln x + (x+2)\left(\dfrac{1}{x}\right) = \dfrac{x\ln x + x + 2}{x}$ and $\quad y' = \dfrac{(x\ln x + x + 2)x^{x+2}}{x}.$

45. $y = (x^2+1)^x$; $\ln y = \ln\,(x^2+1)^x = x\ \ln\,(x^2+1)$. So

$$\frac{y'}{y} = \ln(x^2+1) + x\left(\frac{2x}{x^2+1}\right) = \frac{(x^2+1)\ln(x^2+1)+2x^2}{x^2+1}.$$

$$y' = \frac{[(x^2+1)\ln(x^2+1)+2x^2](x^2+1)^x}{x^2+1}$$

46. $y = x^{\ln x}$; $\ln y = \ln x^{\ln x} = (\ln x)^2$. So $\dfrac{y'}{y} = 2(\ln x)\left(\dfrac{1}{x}\right) = \dfrac{2\ \ln x}{x}$

and $y' = \dfrac{2\ \ln x}{x} \cdot x^{\ln x} = 2(\ln x)x^{\ln x - 1}$.

47. $y = x \ln x$. The slope of the tangent line at any point is

$$y' = \ln x + x\left(\tfrac{1}{x}\right) = \ln x + 1.$$

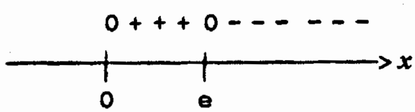
In particular, the slope of the tangent line at (1,0) where $x = 1$ is $m = \ln 1 + 1 = 1$. So, an equation of the tangent line is $y - 0 = 1(x-1)$ or $y = x - 1$.

48. $y = \ln x^2 = 2 \ln x$ and $y' = 2/x$, and this gives the slope of the tangent line at any point (x, y) on the graph of $y = \ln x^2$. In particular, the slope of the tangent line at $(2, \ln 4)$ is $m = 2/2 = 1$. Therefore, the required equation is

$$y - \ln 4 = 1(x-2) \text{ or } y = x + \ln 4 - 2.$$

49. $f(x) = \ln x^2 = 2 \ln x$ and so $f'(x) = 2/x$. Since $f'(x) < 0$ if $x < 0$, and $f'(x) > 0$ if $x > 0$, we see that f is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.

50. $f(x) = \dfrac{\ln x}{x}$. $f'(x) = \dfrac{x\frac{1}{x} - \ln x}{x^2} = \dfrac{1-\ln x}{x^2}$. Observe that $f'(x) = 0$ if $1 - \ln x = 0$ or $x = e$. The sign diagram of f' on $(0,\infty)$



shows that f is increasing on $(0,e)$ and decreasing on (e,∞).

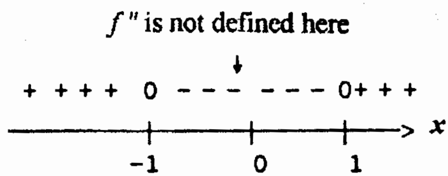
51. $f(x) = x^2 + \ln x^2$; $f'(x) = 2x + \dfrac{2x}{x^2} = 2x + \dfrac{2}{x}$; $f''(x) = 2 - \dfrac{2}{x^2}$.

To find the intervals of concavity for f, we first set $f''(x) = 0$ giving

$$2 - \frac{2}{x^2} = 0, \quad 2 = \frac{2}{x^2}, \quad 2x^2 = 2$$

or $\qquad x^2 = 1$ and $x = \pm 1.$

Next, we construct the sign diagram for f''

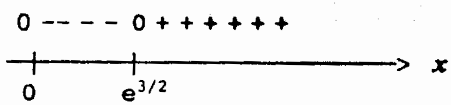


and conclude that f is concave upward on $(-\infty,-1) \cup (1,\infty)$ and concave downward on $(-1,0) \cup (0,1)$.

52. $f(x) = \dfrac{\ln x}{x}$. From Problem 50, we have $f'(x) = \dfrac{1-\ln x}{x^2}$. Then

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1-\ln x)(2x)}{x^4} = \frac{(2\ln x - 3)}{x^3}.$$

Observe that $f''x) = 0$ implies $2 \ln x - 3 = 0$, $\ln x = 3/2$, or $x = e^{3/2}$. From the sign diagram of f'' on $(0,\infty)$,

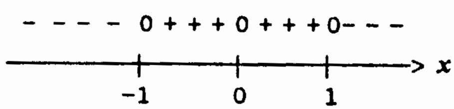


we see that the graph of f is concave downward on $(0,e^{3/2})$ and is concave upward on $(e^{3/2},\infty)$.

53. $f(x) = \ln(x^2 + 1)$. $f'(x) = \dfrac{2x}{x^2+1}$; $f''(x) = \dfrac{(x^2+1)(2)-(2x)(2x)}{(x^2+1)^2} = -\dfrac{2(x^2-1)}{(x^2+1)^2}$.

Setting $f''(x) = 0$ gives $x = \pm 1$ as candidates for inflection points of f.

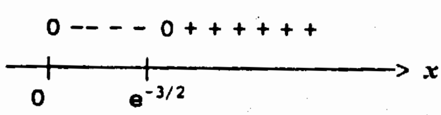
From the sign diagram for f''



we see that $(-1, \ln 2)$ and $(1, \ln 2)$ are inflection points of f.

54. $f(x) = x^2 \ln x$; $f'(x) = 2x\ln x + x^2\left(\frac{1}{x}\right) = 2x\ln x + x$ and

$f''(x) = 2\ln x + 2x\left(\frac{1}{x}\right) + 1 = 2\ln x + 3$. Observe that $f''(x) = 0$ if $2\ln x + 3 = 0$, $\ln x = -3/2$, or $x = e^{-3/2}$. From the sign diagram of f''



we see that $(e^{-3/2}, -\frac{3}{2}e^{-3})$ is an inflection point of f.

55. $f(x) = x - \ln x$; $f'(x) = 1 - \dfrac{1}{x} = \dfrac{x-1}{x} = 0$ if $x = 1$, a critical point of f.

x	1/2	1	3
$f(x)$	1/2 + ln 2	1	3 – ln 3

From the table, we see that f has an absolute minimum at (1,1) and an absolute maximum at (3, 3 – ln 3).