MATH180 – HOMEWORK SOLUTIONS

HOMEWORK #4

Section 3.7: 1, 5, 11, 18, 23

Section 4.1: 1-9 (all), 13, 23, 27, 35, 37, 39, 43, 45-48, 51, 57, 63, 69, 71, 75, 78

Section 4.2: 1-12 (all), 15, 25, 35, 39, 45, 51, 57, 59, 65, 75

Section 4.3: 1, 3, 7, 9, 11-27 (odd only), 33, 35, 41, 45, 53, 63, 65, 69

EXERCISES 3.7, page 241

1. $f(x) = 2x^2$ and $dy = 4x\,dx$.

2. $f(x) = 3x^2 + 1$ and $dy = 6x\,dx$.

3. $f(x) = x^3 - x$ and $dy = (3x^2 - 1)\,dx$.

4. $f(x) = 2x^3 + x$ and $dy = (6x^2 + 1)\,dx$.

5. $f(x) = \sqrt{x+1} = (x+1)^{1/2}$ and $dy = \frac{1}{2}(x+1)^{-1/2}\,dx = \frac{dx}{2\sqrt{x+1}}$.

6. $f(x) = 3x^{-1/2}$ and $dy = -\dfrac{3}{2x^{3/2}}dx.$

7. $f(x) = 2x^{3/2} + x^{1/2}$ and $dy = (3x^{1/2} + \frac{1}{2}x^{-1/2})\,dx = \frac{1}{2}x^{-1/2}(6x+1)dx = \dfrac{6x+1}{2\sqrt{x}}dx.$

8. $f(x) = 3x^{5/6} + 7x^{2/3}$ and $dy = (\frac{5}{2}x^{-1/6} + \frac{14}{3}x^{-1/3})dx.$

9. $f(x) = x + \dfrac{2}{x}$ and $dy = \left(1 - \dfrac{2}{x^2}\right)dx = \dfrac{x^2-2}{x^2}dx.$

10. $f(x) = \dfrac{3}{x-1}$ and $dy = -\dfrac{3}{(x-1)^2}dx.$

11. $f(x) = \dfrac{x-1}{x^2+1}$ and $dy = \dfrac{x^2+1-(x-1)2x}{(x^2+1)^2}dx = \dfrac{-x^2+2x+1}{(x^2+1)^2}dx.$

12. $f(x) = \dfrac{2x^2+1}{x+1}$ and $dy = \dfrac{(x+1)(4x)-(2x^2+1)}{(x+1)^2}dx = \dfrac{2x^2+4x-1}{(x+1)^2}dx.$

13. $f(x) = \sqrt{3x^2-x} = (3x^2-x)^{1/2}$ and

$$dy = \frac{1}{2}(3x^2-x)^{-1/2}(6x-1)dx = \frac{6x-1}{2\sqrt{3x^2-x}}dx.$$

14. $f(x) = (2x^2+3)^{1/3}$ and $dy = \dfrac{1}{3}(2x^2+3)^{-2/3}(4x)dx = \dfrac{4x}{3(2x^2+3)^{2/3}}dx.$

15. $f(x) = x^2 - 1.$
 a. $dy = 2x\,dx.$
 b. $dy \approx 2(1)(0.02) = 0.04.$
 c. $\Delta y = [(1.02)^2 - 1] - [1 - 1] = 0.0404.$

16. $f(x) = 3x^2 - 2x + 6;$
 a. $dy = (6x-2)\,dx$
 b. $dy \approx 10(-0.03) = -0.3.$
 c. $\Delta y = [3(1.97)^2 - 2(1.97) + 6] - [3(2)^2 - 2(2) + 6] = -0.2973.$

17. $f(x) = \dfrac{1}{x}$.

a. $dy = -\dfrac{dx}{x^2}$. b. $dy \approx -0.05$ c. $\Delta y = \dfrac{1}{-0.95} - \dfrac{1}{-1} = -0.05263$.

18. $f(x) = \sqrt{2x+1} = (2x+1)^{1/2}$.

a. $dy = \dfrac{1}{2}(2x+1)^{-1/2}(2) = \dfrac{dx}{\sqrt{2x+1}}$. b. $dy \approx \dfrac{0.1}{\sqrt{9}} = 0.03333$

c. $\Delta y = [2(4.1)+1]^{1/2} - [2(4)+1]^{1/2} = 0.03315$.

19. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{10} = 3 + \dfrac{1}{2\cdot\sqrt{9}} = 3.167$.

20. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{17} = 4 + \dfrac{1}{2\cdot 4} = 4.125$.

21. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{49.5} = 7 + \dfrac{0.5}{2\cdot 7} = 7.0358$.

22. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{99.7} \approx 10 - \dfrac{0.3}{2\cdot 10} = 9.85$.

23. $y = x^{1/3}$ and $dy = \frac{1}{3}x^{-2/3}\,dx$. Therefore, $\sqrt[3]{7.8} = 2 - \dfrac{0.2}{3\cdot 4} = 1.983$.

CHAPTER 4

EXERCISES 4.1, page 262

1. f is decreasing on $(-\infty,0)$ and increasing on $(0, \infty)$.

2. f is decreasing on $(-\infty,-1)$, constant on $(-1,1)$, and increasing on $(1,\infty)$.

3. f is increasing on $(-\infty,-1) \cup (1,\infty)$, and decreasing on $(-1,1)$.

4. f is increasing on $(-\infty,-1) \cup (1,\infty)$ and decreasing on $(-1,0) \cup (0,1)$.

5. f is increasing on $(0,2)$ and decreasing on $(-\infty,0) \cup (2,\infty)$.

6. f is increasing on $(-1,0) \cup (1,\infty)$ and decreasing on $(-\infty,-1) \cup (0,1)$.

7. f is decreasing on $(-\infty,-1) \cup (1,\infty)$ and increasing on $(-1,1)$.

8. f is increasing on $(-\infty,-1) \cup (-1,0) \cup (0,\infty)$.

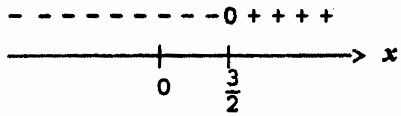
9. Increasing on $(20.2, 20.6) \cup (21.7, 21.8)$, constant on $(19.6, 20.2) \cup (20.6, 21.1)$, and decreasing on $(21.1, 21.7) \cup (21.8, 22.7)$,

10. a. f is decreasing on $(0, 4)$. b. f is constant on $(4, 12)$.
    c. f is increasing on $(12, 24)$.

11. $f(x) = 3x + 5; f'(x) = 3 > 0$ for all x and so f is increasing on $(-\infty,\infty)$.

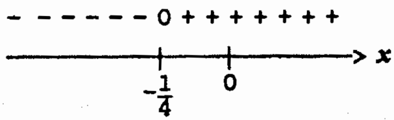
12. $f(x) = 4 - 5x$. $f'(x) = -5$ and, therefore, f is decreasing everywhere, that is, f is decreasing on $(-\infty,\infty)$.

13. $f(x) = x^2 - 3x$. $f'(x) = 2x - 3$ is continuous everywhere and is equal to zero when $x = 3/2$. From the following sign diagram



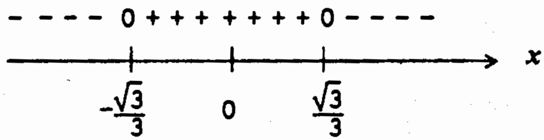
we see that f is decreasing on $(-\infty, \frac{3}{2})$ and increasing on $(\frac{3}{2}, \infty)$.

14. $f(x) = 2x^2 + x + 1$; $f'(x) = 4x + 1 = 0$, if $x = -1/4$. From the sign diagram of f'



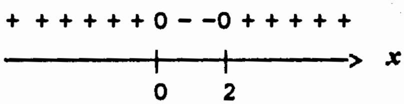
we see that f is decreasing on $(-\infty, -\frac{1}{4})$ and increasing on $(-\frac{1}{4}, \infty)$.

15. $g(x) = x - x^3$. $g'(x) = 1 - 3x^2$ is continuous everywhere and is equal to zero when $1 - 3x^2 = 0$, or $x = \pm\frac{\sqrt{3}}{3}$. From the following sign diagram



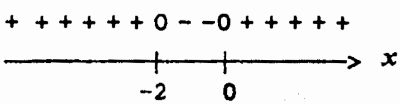
we see that f is decreasing on $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$ and increasing on $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$.

16. $f(x) = x^3 - 3x^2$. $f'(x) = 3x^2 - 6x = 3x(x - 2) = 0$ if $x = 0$ or 2. From the sign diagram of f'



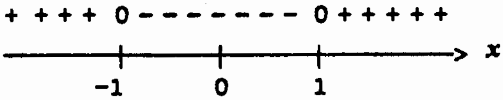
we see that f is increasing on $(-\infty,0) \cup (2,\infty)$ and decreasing on $(0,2)$.

17. $g(x) = x^3 + 3x^2 + 1$; $g'(x) = 3x^2 + 6x = 3x(x + 2)$.
From the following sign diagram



we see that g is increasing on $(-\infty,-2) \cup (0,\infty)$ and decreasing on $(-2,0)$.

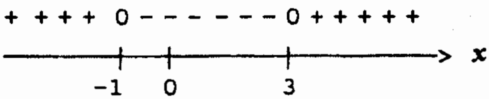
18. $f(x) = x^3 - 3x + 4$. $f'(x) = 3x^2 - 3$ is continuous everywhere and is equal to zero when $x = \pm 1$. From the sign diagram



we see that f is increasing on $(-\infty,-1) \cup (1,\infty)$ and decreasing on $(-1,1)$.

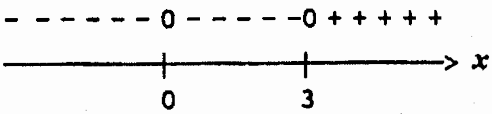
19. $f(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 20$; $f'(x) = x^2 - 6x + 9 = (x - 3)^2 > 0$ for all x except $x = 3$, at which point $f'(3) = 0$. Therefore, f is increasing on $(-\infty,3) \cup (3,\infty)$.

20. $f(x) = \frac{2}{3}x^3 - 2x^2 - 6x - 2$; $f'(x) = 2x^2 - 4x - 6 = 2(x^2 - 2x - 3) = 2(x - 3)(x + 1) = 0$ if $x = -1$ or 3. From the sign diagram of f',



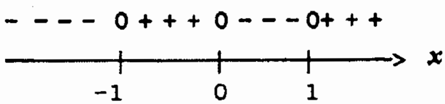
we see that f is increasing on $(-\infty,-1) \cup (3,\infty)$ and decreasing on $(-1,3)$.

21. $h(x) = x^4 - 4x^3 + 10$; $h'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$ if $x = 0$ or 3. From the sign diagram of h',



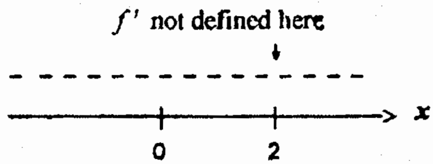
we see that h is increasing on $(3,\infty)$ and decreasing on $(-\infty,0) \cup (0,3)$.

22. $g(x) = x^4 - 2x^2 + 4$. $g'(x) = 4x^3 - 4x = 4x(x^2 - 1)$ is continuous everywhere and is equal to zero when $x = 0$, 1, and -1. From the sign diagram



we see that g is decreasing on $(-\infty,-1) \cup (0,1)$ and increasing on $(-1,0) \cup (1,\infty)$.

23. $f(x)=\frac{1}{x-2}=(x-2)^{-1}$. $f'(x)=-1(x-2)^{-2}(1)=-\frac{1}{(x-2)^2}$ is discontinuous at $x=2$ and is continuous everywhere else. From the sign diagram

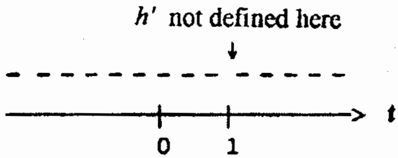


we see that f is decreasing on $(-\infty,2)\cup(2,\infty)$.

24. $h(x)=\frac{1}{2x+3}$; $h'(x)=\frac{-2}{(2x+3)^2}$ and we see that h' is not defined at $x=-3/2$. But $h'(x)<0$ for all x except $x=-3/2$. Therefore, h is decreasing on $(-\infty,-\frac{3}{2})\cup(-\frac{3}{2},\infty)$.

25. $h(t)=\frac{t}{t-1}$. $h'(t)=\frac{(t-1)(1)-t(1)}{(t-1)^2}=-\frac{1}{(t-1)^2}$.

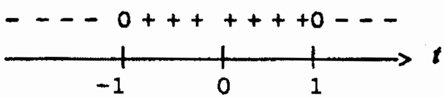
From the following sign diagram,



we see that $h'(t)<0$ whenever it is defined. We conclude that h is decreasing on $(-\infty,1)\cup(1,\infty)$.

26. $g(t)=\frac{2t}{t^2+1}$; $g'(t)=\frac{(t^2+1)(2)-(2t)(2t)}{(t^2+1)^2}=\frac{2t^2+2-4t^2}{(t^2+1)^2}=-\frac{2(t^2-1)}{(t^2+1)^2}$.

Next, $g'(t)=0$ if $t=\pm1$. From the sign diagram of g',



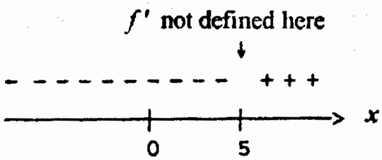
we see that g is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)\cup(1,\infty)$.

27. $f(x) = x^{3/5}$. $f'(x) = \dfrac{3}{5}x^{-2/5} = \dfrac{3}{5x^{2/5}}$. Observe that $f'(x)$ is not defined at $x = 0$, but is positive everywhere else and therefore increasing on $(-\infty,0) \cup (0,\infty)$.

28. $f(x) = x^{2/3} + 5$; $f'(x) = \dfrac{2}{3}x^{-1/3} = \dfrac{2}{3x^{1/3}}$ and so f' is not defined at $x = 0$. Now $f'(x) < 0$ if $x < 0$ and $f'(x) > 0$ if $x > 0$, and so f is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.

29. $f(x) = \sqrt{x+1}$. $f'(x) = \dfrac{d}{dx}(x+1)^{1/2} = \dfrac{1}{2}(x+1)^{-1/2} = \dfrac{1}{2\sqrt{x+1}}$ and we see that $f'(x) > 0$ if $x > -1$. Therefore, f is increasing on $(-1, \infty)$.

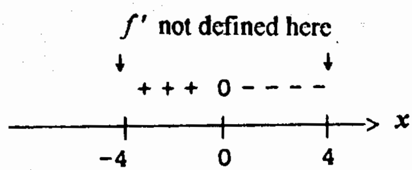
30. $f(x) = (x - 5)^{2/3}$; $f'(x) = \dfrac{2}{3}(x-5)^{-1/3} = \dfrac{2}{3(x-5)^{1/3}}$, and we have the following sign diagram.



So, f is decreasing on $(-\infty,5)$ and is increasing on $(5,\infty)$.

31. $f(x) = \sqrt{16-x^2} = (16-x^2)^{1/2}$. $f'(x) = \dfrac{1}{2}(16-x^2)^{-1/2}(-2x) = -\dfrac{x}{\sqrt{16-x^2}}$.

Since the domain of f is $[-4,4]$, we consider the sign diagram for f' on this interval. Thus,



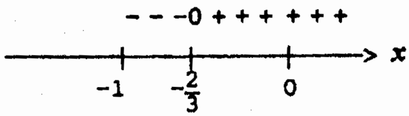
and we see that f is increasing on $(-4,0)$ and decreasing on $(0,4)$.

32. $g(x) = x(x + 1)^{1/2}$.

$$g'(x) = (x+1)^{1/2} + x(\tfrac{1}{2})(x+1)^{-1/2} = (x+1)^{-1/2}(x+1+\tfrac{1}{2}x)$$

$$= (x+1)^{-1/2}(\tfrac{3}{2}x+1) = \frac{3x+2}{2\sqrt{x+1}}.$$

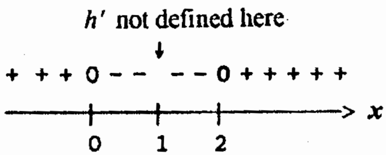
Then g' is continuous on $(-1,\infty)$ and has a zero at $x = -2/3$. From the sign diagram



we see that g is decreasing on $(-1,-\frac{2}{3})$ and increasing on $(-\frac{2}{3},\infty)$.

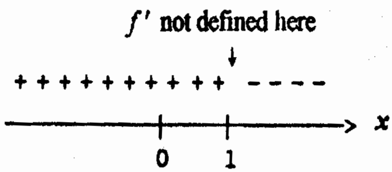
33. $f'(x) = \dfrac{d}{dx}(x - x^{-1}) = 1 + \dfrac{1}{x^2} = \dfrac{x^2+1}{x^2}$ and so $f'(x) > 0$ for all $x \neq 0$.
Therefore, f is increasing on $(-\infty,0) \cup (0,\infty)$.

34. $h(x) = \dfrac{x^2}{x-1}$. $h'(x) = \dfrac{(x-1)(2x) - x^2}{(x-1)^2} = \dfrac{x^2-2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$.
Then, h' is continuous everywhere except at $x = 1$ and has zeros at $x = 0$ and $x = 2$.
From the sign diagram



we see that h is increasing on $(-\infty,0) \cup (2,\infty)$ and decreasing on $(0,1) \cup (1,2)$.

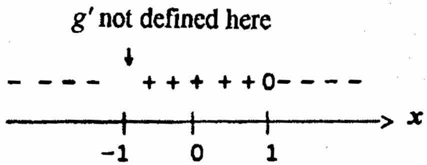
35. $f'(x) = \dfrac{d}{dx}(x-1)^{-2} = -2(x-1)^{-3} = -\dfrac{2}{(x-1)^3}$. From the sign diagram of f'



we see that f is increasing on $(-\infty,1)$ and decreasing on $(1,\infty)$.

36. $g(x) = \dfrac{x}{(x+1)^2}$. $g'(x) = \dfrac{(x+1)^2(1) - x(2)(x+1)}{(x+1)^4} = \dfrac{(x+1)(x+1-2x)}{(x+1)^4} = \dfrac{1-x}{(x+1)^3}$.

So g' has a zero at $x = 1$ and is discontinuous at $x = -1$. The sign diagram of g'



shows that g is decreasing on $(-\infty,-1) \cup (1,\infty)$ and increasing on $(-1,1)$.

37. f has a relative maximum of $f(0) = 1$ and relative minima of $f(-1) = 0$ and $f(1) = 0$.

38. f has a relative maximum of $f(0) = 1$ and relative minima of $f(-1) = 0$ and $f(1) = 0$.

39. f has a relative maximum of $f(-1) = 2$ and a relative minimum of $f(1) = -2$.

40. f has a relative maximum at $x = 0$ with value of 0; it has a relative minimum at (4,-32).

41. f has a relative maximum of $f(1) = 3$ and a relative minimum of $f(2) = 2$.

42. f has a relative minimum at (-1,0).

43. f has a relative minimum at (0,2).

44. f has a relative maximum at $(-3, -\frac{9}{2})$ and a relative minimum at $(3, \frac{9}{2})$.

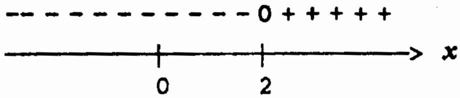
45. a

46. c

47. d

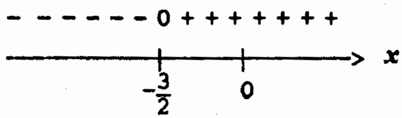
48. b

49. $f(x) = x^2 - 4x$. $f'(x) = 2x - 4 = 2(x - 2)$ has a critical point at $x = 2$. From the following sign diagram



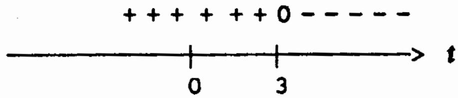
we see that $f(2) = -4$ is a relative minimum by the First Derivative Test.

50. $g(x) = x^2 + 3x + 8$; $g'(x) = 2x + 3$ has a critical point at $x = -3/2$. From the following sign diagram



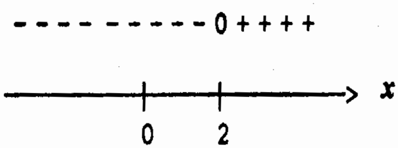
we see that $g(-\frac{3}{2}) = \frac{23}{4}$ is a relative minimum by the First Derivative Test.

51. $h(t) = -t^2 + 6t + 6$; $h'(t) = -2t + 6 = -2(t - 3) = 0$ if $t = 3$, a critical point. The sign diagram



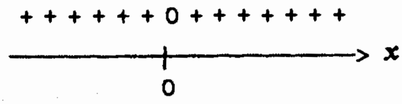
and the First Derivative Test imply that h has a relative maximum at 3 with value $f(3) = -9 + 18 + 6 = 15$.

52. $f(x) = \frac{1}{2}x^2 - 2x + 4$. $f'(x) = x - 2$ giving the critical point $x = 2$. The sign diagram for f' is



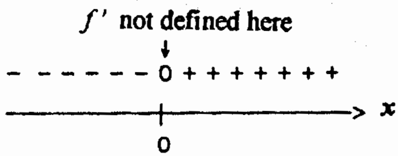
and we see that $f(2) = 2$ is a relative minimum.

53. $f(x) = x^{5/3}$. $f'(x) = \frac{5}{3}x^{2/3}$ giving $x = 0$ as the critical point of f.
From the sign diagram



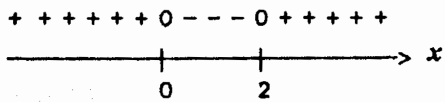
we see that f' does not change sign as we move across $x = 0$, and conclude that f has no relative extremum.

54. $f(x) = x^{2/3} + 2$. $f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$ and is discontinuous at $x = 0$, a critical point. From the sign diagram



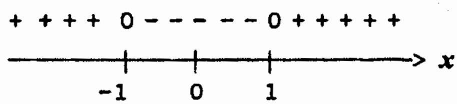
and the First Derivative Test we see that f has a relative minimum at (0,2).

55. $g(x) = x^3 - 3x^2 + 4$. $g'(x) = 3x^2 - 6x = 3x(x - 2) = 0$ if $x = 0$ or 2. From the sign



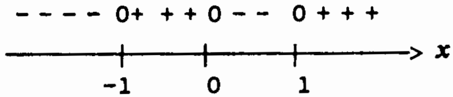
diagram, we see that the critical point $x = 0$ gives a relative maximum, whereas, $x = 2$ gives a relative minimum. The values are $g(0) = 4$ and $g(2) = 8 - 12 + 4 = 0$.

56. $f(x) = x^3 - 3x + 6$. Setting $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x + 1)(x - 1) = 0$ gives $x = -1$ and $x = 1$ as critical points. The sign diagram of f'



shows that (-1,8) is a relative maximum and (1,4) is a relative minimum.

57. $f(x) = \frac{1}{2}x^4 - x^2$. $f'(x) = 2x^3 - 2x = 2x(x^2 - 1) = 2x(x + 1)(x - 1)$ is continuous everywhere and has zeros as $x = -1$, $x = 0$, and $x = 1$, the critical points of f. Using the First Derivative Test and the following sign diagram of f'



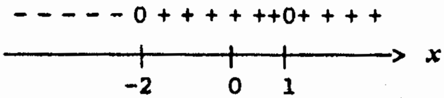
we see that $f(-1) = -1/2$ and $f(1) = -1/2$ are relative minima of f and $f(0) = 0$ is a relative maximum of f.

58. $h(x) = \frac{1}{2}x^4 - 3x^2 + 4x - 8;$

$h'(x) = 2x^3 - 6x + 4 = 2(x^3 - 3x + 2) = 2(x-1)(x^2 + x - 2)$

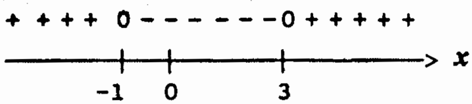
$= 2(x-1)(x-1)(x+2) = 2(x-1)^2(x+2).$

We see that $h'(x) = 0$ at $x = -2$ and $x = 1$; both are critical points of h. From the sign diagram of h'



we see that h has a relative minimum at (-2,-20).

59. $F(x) = \frac{1}{3}x^3 - x^2 - 3x + 4.$ Setting $F'(x) = x^2 - 2x - 3 = (x-3)(x+1) = 0$ gives $x = -1$ and $x = 3$ as critical points. From the sign diagram



we see that $x = -1$ gives a relative maximum and $x = 3$ gives a relative minimum. The values are

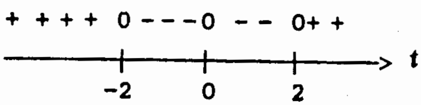
$$F(-1) = -\tfrac{1}{3} - 1 + 3 + 4 = \tfrac{17}{3} \quad \text{and} \quad F(3) = 9 - 9 - 9 + 4 = -5,$$

respectively.

60. $F(t) = 3t^5 - 20t^3 + 20$. Setting

$F'(t) = 15t^4 - 60t^2 = 15t^2(t^2 - 4) = 15t^2(t+2)(t-2) = 0$

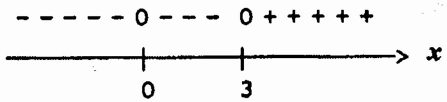
gives $t = -2$, 0, and 2 as critical points. From the sign diagram



we see that $t = -2$ gives a relative maximum and $t = 2$ gives a relative minimum.

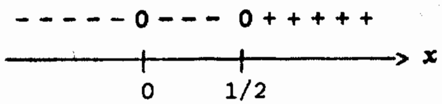
The values are $F(-2) = 3(-32) - 20(-8) + 20 = 84$ and
$F(2) = 3(32) - 20(8) + 20 = -44$, respectively.

61. $g(x) = x^4 - 4x^3 + 8$. Setting $g'(x) = 4x^3 - 12x^2 = 4x^2(x - 3) = 0$ gives $x = 0$ and $x = 3$ as critical points. From the sign diagram



we see that $x = 3$ gives a relative minimum. Its value is $g(3) = 3^4 - 4(3)^3 + 8 = -19$.

62. $f(x) = 3x^4 - 2x^3 + 4$; $f'(x) = 12x^3 - 6x^2 = 6x^2(2x - 1) = 0$ if $x = 0$ or $1/2$. The sign diagram of f' is shown below.



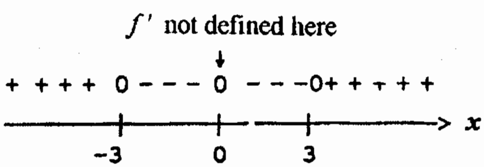
and shows that f has a relative minimum at $(\frac{1}{2}, \frac{63}{16})$.

63. $g'(x) = \dfrac{d}{dx}\left(1 + \dfrac{1}{x}\right) = -\dfrac{1}{x^2}$. Observe that g' is never zero for all values of x.

Furthermore, g' is undefined at $x = 0$, but $x = 0$ is not in the domain of g. Therefore g has no critical points and so g has no relative extrema.

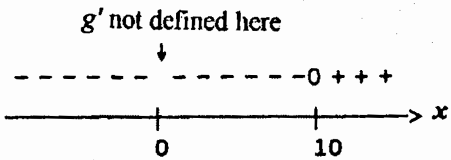
64. $h(x) = \dfrac{x}{x+1}$. $h'(x) = \dfrac{(x+1)(1) - x(1)}{(x+1)^2} = \dfrac{1}{(x+1)^2}$; Since $x = -1$ is not in the domain of h, we see that $x = -1$ is not a critical point of h and conclude that h has no relative extrema.

65. $f(x) = x + \dfrac{9}{x} + 2$. Setting $f'(x) = 1 - \dfrac{9}{x^2} = \dfrac{x^2 - 9}{x^2} = \dfrac{(x+3)(x-3)}{x^2} = 0$ gives $x = -3$ and $x = 3$ as critical points. From the sign diagram



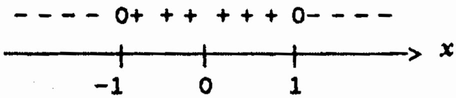
we see that (-3,-4) is a relative maximum and (3,8) is a relative minimum.

66. $g(x)=2x^2+\frac{4000}{x}+10$. $g'(x)=4x-\frac{4000}{x^2}=\frac{4(x^3-1000)}{x^2}$. The only critical point of g is $x=10$; $x=0$ is not a critical point of g since $g(x)$ is not defined there. The sign diagram of g' is



Using the First Derivative Test, we conclude that the point (10, 610) is a relative minimum of g.

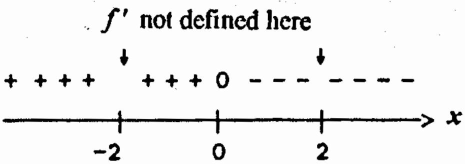
67. $f(x)=\frac{x}{1+x^2}$. $f'(x)=\frac{(1+x^2)(1)-x(2x)}{(1+x^2)^2}=\frac{1-x^2}{(1+x^2)^2}=\frac{(1-x)(1+x)}{(1+x^2)^2}=0$ if $x=\pm1$, and these are critical points of f. From the sign diagram of f'



we see that f has a relative minimum at $(-1,-\frac{1}{2})$ and a relative maximum at $(1,\frac{1}{2})$.

68. $g(x)=\frac{x}{x^2-1}$. Observe that $g'(x)=\frac{(x^2-1)-x(2x)}{(x^2-1)^2}=-\frac{1+x^2}{(x^2-1)^2}$ is never zero. Furthermore, $x\pm1$ are not critical points since they are not in the domain of g. So g has no relative extrema.

69. $f(x)=\frac{x^2}{x^2-4}$. $f'(x)=\frac{(x^2-4)(2x)-x^2(2x)}{(x^2-4)^2}=-\frac{8x}{(x^2-4)^2}$ is continuous everywhere except at $x\pm2$ and has a zero at $x=0$. Therefore, $x=0$ is the only critical point of f (the points $x=\pm2$ do not lie in the domain of f).Using the following sign diagram of f'

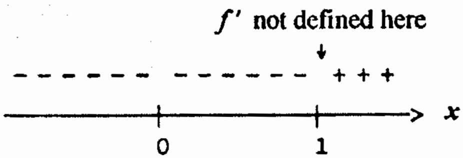


and the First Derivative Test, we conclude that $f(0) = 0$ is a relative maximum of f.

70. $g(t) = \dfrac{t^2}{1+t^2}$. Setting $g'(t) = \dfrac{(1+t^2)(2t) - t^2(2t)}{(1+t^2)^2} = \dfrac{2t}{(1+t^2)^2} = 0$ gives $t = 0$ as the only critical point of g. Since $g'(t) < 0$ if $t < 0$ and $g'(t) > 0$ if $t > 0$, we see that (0,0) is a relative minimum of g.

71. $f(x) = (x - 1)^{2/3}$. $\quad f'(x) = \dfrac{2}{3}(x-1)^{-1/3} = \dfrac{2}{3(x-1)^{1/3}}$.

$f'(x)$ is discontinuous at $x = 1$. The sign diagram for f' is

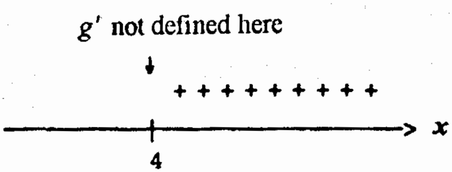


We conclude that $f(1) = 0$ is a relative minimum.

72. $g(x) = x\sqrt{x-4} = x(x-4)^{1/2}$.

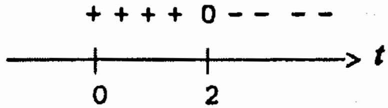
$$g'(x) = (x-4)^{1/2} + x(\tfrac{1}{2})(x-4)^{-1/2} = \tfrac{1}{2}(x-4)^{-1/2}[2(x-4)+x] = \frac{3x-8}{2\sqrt{x-4}}$$

is continuous everywhere except at $x = 4$ and has a zero at $x = 8/3$. Only the point $x = 4$ lies in the domain of g which is the interval $[4,\infty)$. Thus, $x = 4$ is the only critical point of g. From the sign diagram,

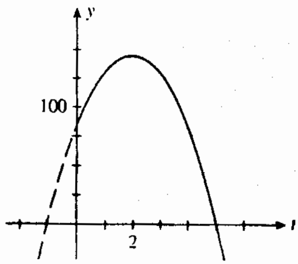


we conclude that $g(4) = 0$ is a relative minimum of g.

73. $h(t) = -16t^2 + 64t + 80$. $\quad h'(t) = -32t + 64 = -32(t - 2)$ and has sign diagram



This tells us that the stone is rising on the time interval (0,2) and falling when $t > 2$. It hits the ground when $h(t) = -16t^2 + 64t + 80 = 0$ or $t^2 - 4t - 5 = (t - 5)(t + 1) = 0$ or $t = 5$ (we reject the root $t = -1$.)



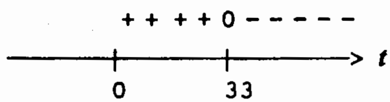
74. $P(x) = -0.001x^2 + 8x - 5000$. $P'(x) = -0.002x + 8 = 0$ if $x = 4000$. Observe that $P'(x) > 0$ if $x < 4000$ and $P'(x) < 0$ if $x > 4000$. So P is increasing on $(0,4000)$ and decreasing on $(4000, \infty)$.

75. $P'(x) = \dfrac{d}{dx}(0.0726x^2 + 0.7902x + 4.9623) = 0.1452x + 0.7902.$

Since $P'(x) > 0$ on (0, 25), we see that P is increasing on the interval in question. Our result tells us that the percent of the population afflicted with Alzheimer's disease increases with age for those that are 65 and over.

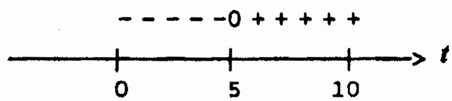
76. $h(t) = -\frac{1}{3}t^3 + 16t^2 + 33t + 10$; $h'(t) = -t^2 + 32t + 33 = -(t + 1)(t - 33)$.

The sign diagram for h' is



The rocket is rising on the time interval (0,33) and descending on $(33,T)$ for some positive number T. The parachute is deployed 33 seconds after liftoff.

77. $I(t) = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 80$; $I'(t) = t^2 - 5t = t(t - 5) = 0$ if $t = 0$ or 5. From the sign

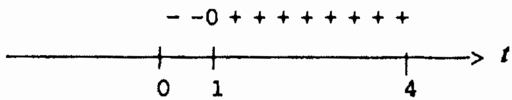


diagram, we see that I is decreasing on (0,5) and increasing on (5,10). After declining from 1984 through 1989, the index begins to increase after 1989.

78. $f(t) = 20t - 40\sqrt{t} + 50 = 20t - 40t^{1/2} + 50.$

$$f'(t) = 20 - 40(\frac{1}{2}t^{-1/2}) = 20(1 - \frac{1}{\sqrt{t}}) = \frac{20(\sqrt{t} - 1)}{\sqrt{t}}.$$

Then f' is continuous on (0,4) and is equal to zero at $t = 1$. From the sign diagram



we see that f is decreasing on (0,1) and increasing on (1,4). We conclude that the average speed decreases from 6 A.M. to 7 A.M. and then picks up from 7 A.M. to 10 A. M.

EXERCISES 4.2, page 280

1. f is concave downward on $(-\infty,0)$ and concave upward on $(0,\infty)$. f has an inflection point at (0,0).

2. f is concave downward on $(0, \frac{3}{2})$ and concave upward on $(\frac{3}{2}, \infty)$. f has an inflection point at $(\frac{3}{2}, 2)$.

3. f is concave downward on $(-\infty,0) \cup (0,\infty)$.

4. f is concave upward on $(-\infty,-4) \cup (4,\infty)$; concave downward on $(-4,4)$.

5. f is concave upward on $(-\infty,0) \cup (1,\infty)$ and concave downward on $(0,1)$. (0,0) and (1,-1) are inflection points of f.

6. f is concave upward on $(0,1) \cup (5,\infty)$ and concave downward on $(1,5)$.

7. f is concave downward on $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$.

8. f is concave downward on $(-\infty,0)$ and concave upward on $(0,\infty)$. $(0,1)$ is an inflection point.

9. a   10. b   11. b   12. c

13. a. $D_1'(t) > 0$, $D_2'(t) > 0$, $D_1''(t) > 0$, and $D_2''(t) < 0$ on $(0,12)$.
b. With or without the proposed promotional campaign, the deposits will increase, but with the promotion, the deposits will increase at an increasing rate whereas without the promotion, the deposits will increase at a decreasing rate.

14. If you look at the tangent lines to the graph of P, you will see that the tangent line at P has the greatest slope. This means that the rate at which the average worker is assembling transistor radios is the greatest-- that is, she is most efficient-- at $t = 2$, or at 10 A.M.

15. The significance of the inflection point Q is that the restoration process is working at its peak at the time t_0 corresponding to its t-coordinate.

16. The rumor spreads with increasing speed initially. The rate at which the rumor is spread reaches a maximum at the time corresponding to the t-coordinate of the point P on the curve. Thereafter, the speed at which the rumor is spread decreases.

17. $f(x) = 4x^2 - 12x + 7$. $f'(x) = 8x - 12$ and $f''(x) = 8$. So, $f''(x) > 0$ everywhere and therefore f is concave upward everywhere.

18. $g(x) = x^4 + \frac{1}{2}x^2 + 6x + 10$; $g'(x) = 4x^3 + x + 6$ and $g''(x) = 12x^2 + 1$. We see that $g''(x) \geq 1$ for all values of x and so g is concave upward everywhere.

19. $f(x) = \dfrac{1}{x^4} = x^{-4}$; $f'(x) = -\dfrac{4}{x^5}$ and $f''(x) = \dfrac{20}{x^6} > 0$ for all values of x in $(-\infty,0) \cup (0,\infty)$ and so f is concave upward everywhere.

20. $g(x) = -\sqrt{4-x^2}$. $g'(x) = \frac{d}{dx}\left[-(4-x^2)^{1/2}\right] = -\frac{1}{2}(4-x^2)^{-1/2}(-2x) = x(4-x^2)^{-1/2}$.

$$g''(x) = (4-x^2)^{-1/2} + x(-\tfrac{1}{2})(4-x^2)^{-3/2}(-2x)$$

$$= (4-x^2)^{-3/2}[(4-x^2)+x^2] = \frac{4}{(4-x^2)^{3/2}} > 0,$$

whenever it is defined and so g is concave upward wherever it is defined.

21. $f(x) = 2x^2 - 3x + 4$; $f'(x) = 4x - 3$ and $f''(x) = 4 > 0$ for all values of x. So f is concave upward on $(-\infty,\infty)$.

22. $g(x) = -x^2 + 3x + 4$; $g'(x) = -2x + 3$ and $g''(x) = -2 < 0$ for all values of x. So g is concave downward on $(-\infty,\infty)$.

23. $f(x) = x^3 - 1$. $f'(x) = 3x^2$ and $f''(x) = 6x$. The sign diagram of f'' follows.

```
- - - - - - - 0 + + + + + + +
--------------+---------------> x
              0
```

We see that f is concave downward on $(-\infty,0)$ and concave upward on $(0,\infty)$.

24. $g(x) = x^3 - x$. $g'(x) = 3x^2 - 1$ and $g''(x) = 6x$. Since $g''(x) < 0$ if $x < 0$ and $g''(x) > 0$ if $x > 0$, we see that g is concave downward on $(-\infty,0)$ and concave upward on $(0,\infty)$.

25. $f(x) = x^4 - 6x^3 + 2x + 8$; $f'(x) = 4x^3 - 18x^2 + 2$ and $f''(x) = 12x^2 - 36x = 12x(x - 3)$. The sign diagram of f''

```
+ + + + + + 0 - - - - 0 + + +
------------+---------+-------> x
            0         3
```

shows that f is concave upward on $(-\infty,0) \cup (3,\infty)$ and concave downward on $(0,3)$.

26. $f(x) = 3x^4 - 6x^3 + x - 8$. $f'(x) = 12x^3 - 18x^2 + 1$ and $f''(x) = 36x^2 - 36x = 36x(x - 1)$. From the sign diagram of f''

```
+ + + + + + 0 - - 0 + + + + +
------------+-----+-----------> x
            0     1
```

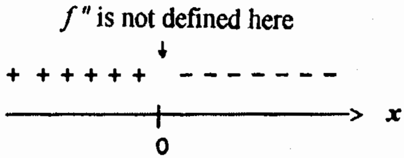
we conclude that f is concave upward on $(-\infty,0) \cup (1,\infty)$ and concave downward on $(0,1)$.

27. $f(x)=x^{4/7}$. $f'(x)=\frac{4}{7}x^{-3/7}$ and $f''(x)=-\frac{12}{49}x^{-10/7}=-\frac{12}{49x^{10/7}}$.

Observe that $f''(x)<0$ for all x different from zero. So f is concave downward on $(-\infty,0)\cup(0,\infty)$.

28. $f(x)=x^{1/3}$; $f'(x)=\frac{1}{3}x^{-2/3}$ and $f''(x)=-\frac{2}{9}x^{-5/3}=-\frac{2}{9x^{5/3}}$.

From the sign diagram of f'',



we see that f is concave upward on $(-\infty,0)$ and concave downward on $(0,\infty)$.

29. $f(x)=(4-x)^{1/2}$. $f'(x)=\frac{1}{2}(4-x)^{-1/2}(-1)=-\frac{1}{2}(4-x)^{-1/2}$;

$$f''(x)=\frac{1}{4}(4-x)^{-3/2}(-1)=-\frac{1}{4(4-x)^{3/2}}<0.$$

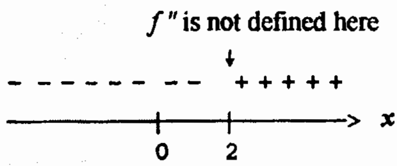
whenever it is defined. So f is concave downward on $(-\infty,4)$.

30. $g(x)=\sqrt{x-2}=(x-2)^{1/2}$. $g'(x)=\frac{1}{2}(x-2)^{-1/2}$

and $g''(x)=-\frac{1}{4}(x-2)^{-3/2}=-\frac{1}{4(x-2)^{3/2}}$, which is negative for $x>2$. Next, the domain of g is $[2,\infty)$, and we conclude that g is concave downward on $(2,\infty)$.

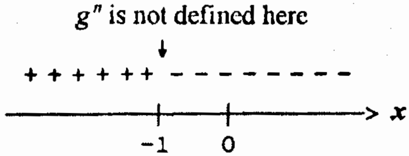
31. $f'(x)=\frac{d}{dx}(x-2)^{-1}=-(x-2)^{-2}$ and $f''(x)=2(x-2)^{-3}=\frac{2}{(x-2)^3}$.

The sign diagram of f'' shows that f is concave downward on $(-\infty,2)$ and concave upward on $(2,\infty)$.



32. $g(x)=\dfrac{x}{x+1}$; $g'(x)=\dfrac{(x+1)(1)-x(1)}{(x+1)^2}=\dfrac{1}{(x+1)^2}=(x+1)^{-2}$ and

$g''(x)=-2(x+1)^{-3}=-\dfrac{2}{(x+1)^3}$. The sign diagram of $g''(x)$ is



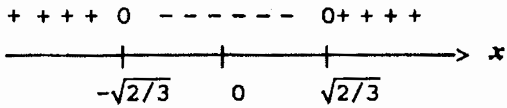
and we see that g is concave upward on $(-\infty,-1)$ and concave downward on $(-1,\infty)$.

33. $f'(x)=\dfrac{d}{dx}(2+x^2)^{-1}=-(2+x^2)^{-2}(2x)=-2x(2+x^2)^{-2}$ and

$f''(x)=-2(2+x^2)^{-2}-2x(-2)(2+x^2)^{-3}(2x)$

$=2(2+x^2)^{-3}[-(2+x^2)+4x^2]=\dfrac{2(3x^2-2)}{(2+x^2)^3}=0$ if $x=\pm\sqrt{2/3}$.

From the sign diagram of f''



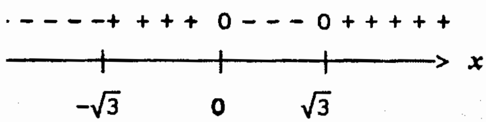
we see that f is concave upward on $(-\infty,-\sqrt{2/3})\cup(\sqrt{2/3},\infty)$ and concave downward on $(-\sqrt{2/3},\sqrt{2/3})$.

34. $g(x)=\dfrac{x}{1+x^2}$; $g'(x)=\dfrac{(1+x^2)(1)-x(2x)}{(1+x^2)^2}=\dfrac{1-x^2}{(1+x^2)^2}$

$g''(x)=\dfrac{(1+x^2)^2(-2x)-(1-x^2)2(1+x^2)(2x)}{(1+x^2)^4}$

$=\dfrac{-2x(1+x^2)(1+x^2+2-2x^2)}{(1+x^2)^4}=-\dfrac{2x(3-x^2)}{(1+x^2)^3}$.

The sign diagram for g'' follows:



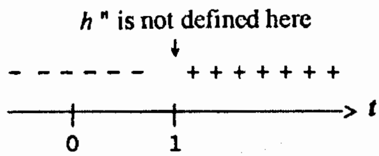
We see that g is concave downward on $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$ and concave upward on $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$.

35. $h(t) = \dfrac{t^2}{t-1}$; $h'(t) = \dfrac{(t-1)(2t) - t^2(1)}{(t-1)^2} = \dfrac{t^2 - 2t}{(t-1)^2}$;

$$h''(t) = \frac{(t-1)^2(2t-2) - (t^2-2t)2(t-1)}{(t-1)^4}$$

$$= \frac{(t-1)(2t^2 - 4t + 2 - 2t^2 + 4t)}{(t-1)^4} = \frac{2}{(t-1)^3}.$$

The sign diagram of h'' is

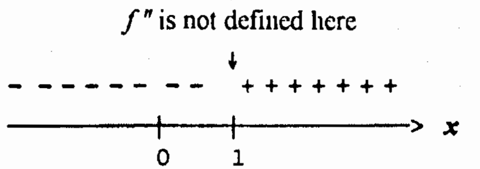


and tells us that h is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$.

36. $f(x) = \dfrac{x+1}{x-1}$; $f'(x) = \dfrac{(x-1)(1) - (x+1)(1)}{(x-1)^2} = -\dfrac{2}{(x-1)^2} = -2(x-1)^{-2}$ and

$$f''(x) = (-2)(-2)(x-1)^{-3} = \frac{4}{(x-1)^3}.$$

The sign diagram of f'' is



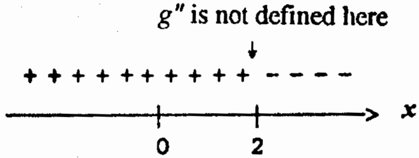
and we conclude that f is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$.

37. $g(x) = x + \dfrac{1}{x^2}$. $g'(x) = 1 - 2x^{-3}$ and $g''(x) = 6x^{-4} = \dfrac{6}{x^4} > 0$ whenever $x \neq 0$.
Therefore, g is concave upward on $(-\infty,0) \cup (0,\infty)$.

38. $h(r) = -(r-2)^{-2}$; $h'(r) = 2(r-2)^{-3}$; $h''(r) = -6(r-2)^{-4} < 0$ for all $r \neq 2$.
So h is concave downward on $(-\infty,2) \cup (2,\infty)$.

39. $g(t) = (2t - 4)^{1/3}$. $g'(t) = = \frac{1}{3}(2t-4)^{-2/3}(2) = \frac{2}{3}(2t-4)^{-2/3}$.

$g''(t) = -\frac{4}{9}(2t-4)^{-5/3} = -\frac{4}{9(2t-4)^{5/3}}$. The sign diagram of g''



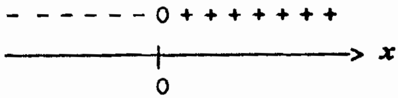
tells us that g is concave upward on $(-\infty,2)$ and concave downward on $(2,\infty)$.

40. $f(x) = (x - 2)^{2/3}$.

$f'(x) = \frac{2}{3}(x-2)^{-1/3}$ and $f''(x) = -\frac{2}{9}(x-2)^{-4/3} = -\frac{2}{9(x-2)^{4/3}} < 0$

for all $x \neq 2$. Therefore, f is concave downward on $(-\infty,2) \cup (2,\infty)$.

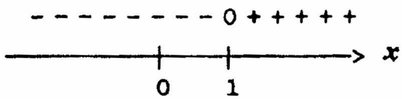
41. $f(x) = x^3 - 2$. $f'(x) = 3x^2$ and $f''(x) = 6x$. $f''(x)$ is continuous everywhere and has a zero at $x = 0$. From the sign diagram of f''



we conclude that $(0,-2)$ is an inflection point of f.

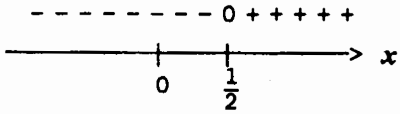
42. $g(x) = x^3 - 6x$. $g'(x) = 3x^2 - 6$ and $g''(x) = 6x$. Observe that $g''(x) = 0$ if $x = 0$. Since $g''(x) < 0$ if $x < 0$ and $g''(x) > 0$ if $x > 0$, we see that $(0,0)$ is an inflection point of g.

43. $f(x) = 6x^3 - 18x^2 + 12x - 15$; $f'(x) = 18x^2 - 36x + 12$ and $f''(x) = 36x - 36 = 36(x - 1) = 0$ if $x = 1$. The sign diagram of f''



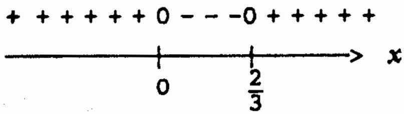
tells us that f has an inflection point at $(1,-15)$.

44. $g(x) = 2x^3 - 3x^2 + 18x - 8$, $g'(x) = 6x^2 - 6x + 18$ and $g''(x) = 12x - 6 = 6(2x - 1)$. From the sign diagram of g''



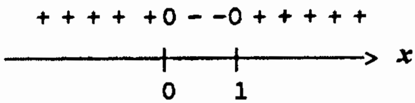
we conclude that $(\frac{1}{2}, \frac{1}{2})$ is an inflection point of g.

45. $f(x) = 3x^4 - 4x^3 + 1$. $f'(x) = 12x^3 - 12x^2$ and $f''(x) = 36x^2 - 24x = 12x(3x - 2) = 0$ if $x = 0$ or $2/3$. These are candidates for inflection points. The sign diagram of f''



shows that (0,1) and $(\frac{2}{3}, \frac{11}{27})$ are inflection points of f.

46. $f(x) = x^4 - 2x^3 + 6$. $f'(x) = 4x^3 - 6x^2$ and $f''(x) = 12x^2 - 12x = 12x(x - 1)$. $f''(x)$ is continuous everywhere and has zeros at $x = 0$ and $x = 1$. From the sign diagram of f''



we conclude that (0,6) and (1,5) are inflection points of f.

47. $g(t) = t^{1/3}$, $g'(t) = \frac{1}{3}t^{-2/3}$ and $g''(t) = -\frac{2}{9}t^{-5/3} = -\dfrac{2}{9t^{5/3}}$. Observe that $t = 0$ is in the domain of g. Next, since $g''(t) > 0$ if $t < 0$ and $g''(t) < 0$, if $t > 0$, we see that (0,0) is an inflection point of g.

48. $f(x) = x^{1/5}$. $f'(x) = \frac{1}{5}x^{-4/5}$ and $f''(x) = -\frac{4}{25}x^{-9/5} = -\dfrac{4}{25x^{9/5}}$. Observe that $f''(x) > 0$ if $x < 0$ and $f''(x) < 0$ if $x > 0$. Therefore, (0,0) is an inflection point.

49. $f(x) = (x - 1)^3 + 2$. $f'(x) = 3(x - 1)^2$ and $f''(x) = 6(x - 1)$. Observe that $f''(x) < 0$ if $x < 1$ and $f''(x) > 0$ if $x > 1$ and so (1,2) is an inflection point of f.

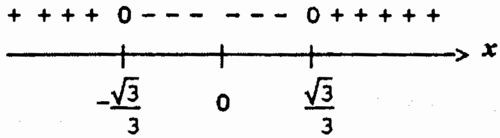
50. $f(x) = (x-2)^{4/3}$. $f'(x) = \frac{4}{3}(x-2)^{1/3}$. $f''(x) = \frac{4}{9}(x-2)^{-2/3} = \dfrac{4}{9(x-2)^{2/3}}$.
$x = 2$ is a candidate for an inflection point of f, but $f''(x) > 0$ for all values of $x = 2$ and so f has no inflection point.

51. $f(x) = \dfrac{2}{1+x^2} = 2(1+x^2)^{-1}$. $f'(x) = -2(1+x^2)^{-2}(2x) = -4x(1+x^2)^{-2}$.

$$f''(x) = -4(1+x^2)^{-2} - 4x(-2)(1+x^2)^{-3}(2x)$$

$$= 4(1+x^2)^{-3}[-(1+x^2)+4x^2] = \frac{4(3x^2-1)}{(1+x^2)^3},$$

is continuous everywhere and has zeros at $x = \pm\frac{\sqrt{3}}{3}$. From the sign diagram of f'' we conclude that $(-\frac{\sqrt{3}}{3}, \frac{3}{2})$ and $(\frac{\sqrt{3}}{3}, \frac{3}{2})$ are inflection points of f.



52. $f(x) = 2 + \dfrac{3}{x}$. $f'(x) = -\dfrac{3}{x^2}$ and $f''(x) = \dfrac{6}{x^3}$. Now f'' changes sign as we move across $x = 0$ but $x = 0$ is not in the domain of f so f has no inflection points.

53. $f(x) = -x^2 + 2x + 4$ and $f'(x) = -2x + 2$. The critical point of f is $x = 1$. Since $f''(x) = -2$ and $f''(1) = -2 < 0$, we conclude that $f(1) = 5$ is a relative maximum of f.

54. $g(x) = 2x^2 + 3x + 7$; $g'(x) = 4x + 3 = 0$ if $x = -3/4$ and this is a critical point of g. Next, $g''(x) = 4$ and so $g''(-\frac{3}{4}) = 4 > 0$. So $(-\frac{3}{4}, \frac{47}{8})$ is a relative minimum.

55. $f(x) = 2x^3 + 1$; $f'(x) = 6x^2 = 0$ if $x = 0$ and this is a critical point of f. Next, $f''(x) = 12x$ and so $f''(0) = 0$. Thus, the Second Derivative Test fails. But the First Derivative Test shows that (0,0) is not a relative extremum.

56. $g(x) = x^3 - 6x$. $g'(x) = 3x^2 - 6 = 3(x^2 - 2) = 0$ implies $x = \pm\sqrt{2}$, are the critical points of g. Next, $g''(x) = 6x$. Since $g''(-\sqrt{2}) = -6\sqrt{2} < 0$ and $g''(\sqrt{2}) = 6\sqrt{2} > 0$, we conclude, by the Second Derivative Test, that $(-\sqrt{2}, 4\sqrt{2})$ is a relative maximum and $(\sqrt{2}, -4\sqrt{2})$ is a relative minimum of g.

57. $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 10$. $f'(x) = x^2 - 4x - 5 = (x - 5)(x + 1)$ and this gives $x = -1$ and $x = 5$ as critical points of f. Next, $f''(x) = 2x - 4$. Since $f''(-1) = -6 < 0$, we see that $(-1, -\frac{22}{3})$ is a relative maximum. Next, $f''(5) = 6 > 0$ and this shows that $(5, -\frac{130}{3})$ is a relative minimum.

58. $f(x) = 2x^3 + 3x^2 - 12x - 4$; $f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x + 2)(x - 1)$. The critical points of f are $x = -2$ and $x = 1$. $f''(x) = 12x + 6 = 6(2x + 1)$. Then $f''(-2) = 6(-4 + 1) = -18 < 0$ and $f''(1) = 6(2 + 1) = 18 > 0$. Using the Second Derivative Test, we conclude that $f(-2) = 16$ is a relative maximum and $f(1) = -11$ is a relative minimum.

59. $g(t) = t + \dfrac{9}{t}$. $g'(t) = 1 - \dfrac{9}{t^2} = \dfrac{t^2 - 9}{t^2} = \dfrac{(t+3)(t-3)}{t^2}$ and this shows that $t = \pm 3$ are critical points of g. Now, $g''(t) = 18t^{-3} = \dfrac{18}{t^3}$. Since $g''(-3) = -\dfrac{18}{27} < 0$ the Second Derivative Test implies that g has a relative maximum at (-3,-6). Also, $g''(3) = \dfrac{18}{27} > 0$ and so g has a relative minimum at (3,6).

60. $f(t) = 2t + 3t^{-1}$. $f'(t) = 2 - 3t^{-2}$. Setting $f'(t) = 0$ gives $3t^{-2} = 2$ or $t^2 = 3/2$, so that $t = \pm\sqrt{3/2}$ are critical points of f. Next, we compute $f''(t) = 6/t^3$. Since $f''(-\sqrt{3/2}) < 0$ and $f''(\sqrt{3/2}) > 0$, we see that $f(-\sqrt{3/2}) = -2\sqrt{3/2} - 3\sqrt{2/3}$ is a relative maximum and $f(\sqrt{3/2}) = 2\sqrt{3/2} + 3\sqrt{2/3}$ is a relative minimum of f.

61. $f(x) = \dfrac{x}{1-x}$. $f'(x) = \dfrac{(1-x)(1) - x(-1)}{(1-x)^2} = \dfrac{1}{(1-x)^2}$ is never zero.

So there are no critical points and f has no relative extrema.

62. $f(x) = \dfrac{2x}{x^2+1}$. $f'(x) = \dfrac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2} = \dfrac{2(1-x^2)}{(x^2+1)^2} = 0$ if $x = \pm 1$.

So $x = \pm 1$ are critical points of f.

$$\text{Next, } f''(x) = \frac{(x^2+1)^2(-4x) - 2(1-x^2)2(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{2x(x^2+1)(-2x^2 - 2 - 4 + 4x^2)}{(x^2+1)^4} = \frac{4x(x^2-3)}{(x^2+1)^3}.$$

Since $f''(-1) = \dfrac{-2(-4)}{2^3} = 1 > 0$, we see that (-1,-1) is a relative minimum and

$f''(-1) = \frac{-2(-4)}{2^3} = 1 < 0,$ we see that (1,1) is a relative maximum.

63. $f(t) = t^2 - \frac{16}{t}.$ $f'(t) = 2t + \frac{16}{t^2} = \frac{2t^3+16}{t^2} = \frac{2(t^3+8)}{t^2}.$ Setting $f'(t) = 0$ gives $t = -2$ as a critical point. Next, we compute $f''(t) = \frac{d}{dt}(2t + 16t^{-2}) = 2 - 32t^{-3} = 2 - \frac{32}{t^3}.$ Since $f''(-2) = 2 - \frac{32}{(-8)} = 6 > 0,$ we see that (-2,12) is a relative minimum.

64. $g(x) = x^2 + \frac{2}{x}. g'(x) = 2x - \frac{2}{x^2}.$ Setting $g'(x) = 0$ gives $x^3 = 1$ or $x = 1$. Thus, $x = 1$ is the only critical point of g. Next, $g''(x) = 2 + \frac{4}{x^3}.$ Since $g''(1) = 6 > 0$, we conclude that $g(1) = 3$ is a relative minimum of g.

65. $g(s) = \frac{s}{1+s^2}; g'(s) = \frac{(1+s^2)(1) - s(2s)}{(1+s^2)^2} = \frac{1-s^2}{(1+s^2)^2} = 0$ gives $s = -1$ and $s = 1$ as critical points of g. Next, we compute

$$g''(s) = \frac{(1+s^2)^2(-2s) - (1-s^2)2(1+s^2)(2s)}{(1+s^2)^4}$$

$$= \frac{2s(1+s^2)(-1-s^2-2+2s^2)}{(1+s^2)^4} = \frac{2s(s^2-3)}{(1+s^2)^3}.$$

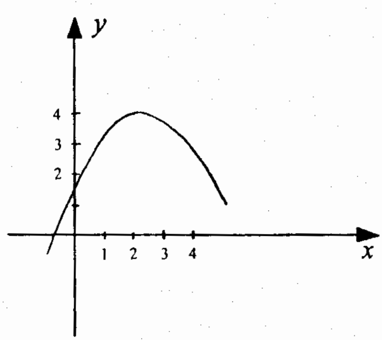
Now, $g''(-1) = \frac{1}{2} > 0$ and so $g(-1) = -\frac{1}{2}$ is a relative minimum of g. Next, $g''(1) = -\frac{1}{2} < 0$ and so $g(1) = \frac{1}{2}$ is a relative maximum of g.

66. $g'(x) = \frac{d}{dx}(1+x^2)^{-1} = -(1+x^2)^{-2}(2x) = -\frac{2x}{(1+x^2)^2}.$ Setting $g'(x) = 0$ gives $x = 0$ as the only critical point. Next, we find

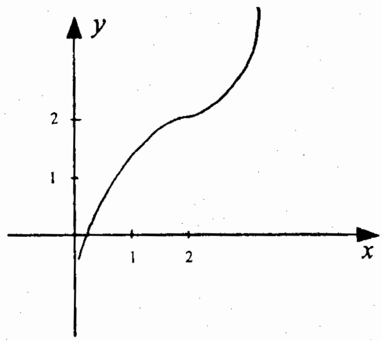
$$g'(x) = \frac{(1+x^2)^2(-2) + 2x(2)(1+x^2)(2x)}{(1+x^2)^4} = \frac{-2(1+x^2)(1+x^2-4x^2)}{(1+x^2)^4} = -\frac{2(1-3x^2)}{(1+x^2)^3}.$$

Since $g''(0) = -2 < 0$, we see that (0,1) is a relative maximum.

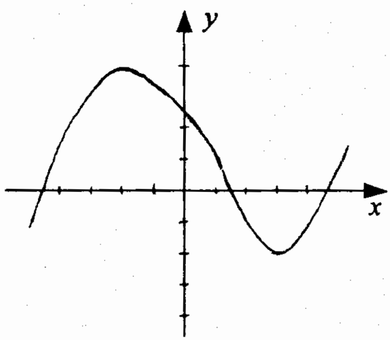
69.



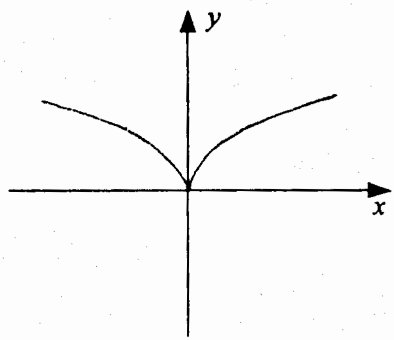
70.



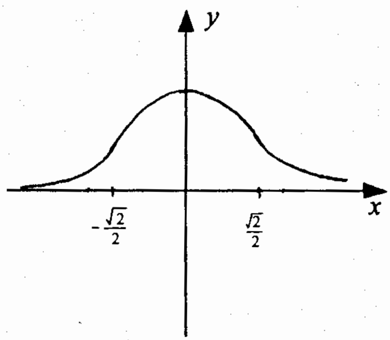
71.



72.



73.



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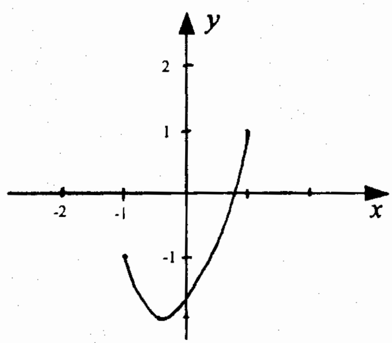


75. a. $N'(t)$ is positive because N is increasing on $(0,12)$.

b. $N''(t) < 0$ on $(0,6)$ and $N''(t) > 0$ on $(6,12)$.

c. The rate of growth of the number of help-wanted advertisements was decreasing over the first six months of the year and increasing over the last six months.

EXERCISES 4.3, page 296

1. $y = 0$ is a horizontal asymptote.

2. $y = 0$ is a horizontal asymptote and $x = -1$ is a vertical asymptote.

3. $y = 0$ is a horizontal asymptote and $x = 0$ is a vertical asymptote.

4. $y = 0$ is a horizontal asymptote.

5. $y = 0$ is a horizontal asymptote and $x = -1$ and $x = 1$ are vertical asymptotes.

6. $y = 0$ is a horizontal asymptote.

7. $y = 3$ is a horizontal asymptote and $x = 0$ is a vertical asymptote.

8. $y = 0$ is a horizontal asymptote, and $x = -2$ is a vertical asymptote.

9. $y = 1$ and $y = -1$ are horizontal asymptotes.

10. $y = 1$ is a horizontal asymptote and $x = \pm 1$ are vertical asymptotes.

11. $\lim_{x\to\infty} \frac{1}{x} = 0$ and so $y = 0$ is a horizontal asymptote. Next, since the numerator of the rational expression is not equal to zero and the denominator is zero at $x = 0$, we see that $x = 0$ is a vertical asymptote.

12. $\lim_{x\to\infty} \frac{1}{x+2} = 0$ and so $y = 0$ is a horizontal asymptote, Next, observe that the numerator of the rational function is not equal to zero but the denominator is equal to zero at $x = -2$ and so $x = -2$ is a vertical asymptote.

13. $f(x) = -\frac{2}{x^2}$. $\lim_{x\to\infty} -\frac{2}{x^2} = 0$, so $y = 0$ is a horizontal asymptote. Next, the denominator of $f(x)$ is equal to zero at $x = 0$. Since the numerator of $f(x)$ is not

equal to zero at $x = 0$, we see that $x = 0$ is a vertical asymptote.

14. $\lim_{x\to\infty} \frac{1}{1+2x^2} = 0$ and so $y = 0$ is a horizontal asymptote, Next, observe that the denominator $1 + 2x^2 \neq 0$ and so there are no vertical asymptotes.

15. $\lim_{x\to\infty} \frac{x-1}{x+1} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = 1$, and so $y = 1$ is a horizontal asymptote. Next, the denominator is equal to zero at $x = -1$ and the numerator is not equal to zero at this point, so $x = -1$ is a vertical asymptote.

16. $\lim_{t\to\infty} \frac{t+1}{2t-1} = \lim_{t\to\infty} \frac{1+\frac{1}{t}}{2-\frac{1}{t}} = \frac{1}{2}$, and so $y = 1/2$ is a horizontal asymptote. Next, observe that the denominator of the rational expression is zero at $t = 1/2$, but the numerator is not equal to zero at this point, and so $t = 1/2$ is a vertical asymptote.

17. $h(x) = x^3 - 3x^2 + x + 1$. $h(x)$ is a polynomial function and, therefore, it does not have any horizontal or vertical asymptotes.

18. The function g is a polynomial, and so the graph of g has no horizontal or vertical asymptotes.

19. $\lim_{t\to\infty} \frac{t^2}{t^2-9} = \lim_{t\to\infty} \frac{1}{1-\frac{9}{t^2}} = 1$, and so $y = 1$ is a horizontal asymptote. Next, observe that the denominator of the rational expression $t^2 - 9 = (t+3)(t-3) = 0$ if $t = -3$ and $t = 3$. But the numerator is not equal to zero at these points. Therefore, $t = -3$ and $t = 3$ are vertical asymptotes.

20. $\lim_{x\to\infty} \frac{x^3}{x^2-4} = \lim_{x\to\infty} \frac{x}{1-\frac{4}{x^2}} = \infty$, and, similarly, $\lim_{x\to\infty} \frac{x^3}{x^2-4} = -\infty$. Therefore, there are no horizontal asymptotes. Next, note that the denominator of $g(x)$ equals zero at $x \pm 2$. Since the numerator of $g(x)$ is not equal to zero at $x \pm 2$, we see that $x = -2$ and $x = 2$ are vertical asymptotes.

21. $\lim_{x\to\infty}\frac{3x}{x^2-x-6}=\lim_{x\to\infty}\frac{\frac{3}{x}}{1-\frac{1}{x}-\frac{6}{x^2}}=0$ and so $y=0$ is a horizontal asymptote. Next, observe that the denominator $x^2-x-6=(x-3)(x+2)=0$ if $x=-2$ or $x=3$. But the numerator $3x$ is not equal to zero at these points. Therefore, $x=-2$ and $x=3$ are vertical asymptotes.

22. $\lim_{x\to\infty}\frac{2x}{x^2+x-2}=\lim_{x\to\infty}\frac{\frac{2}{x}}{1+\frac{1}{x}-\frac{2}{x^2}}=0$, and so $y=0$ is a horizontal asymptote. Next, observe that the denominator $x^2+x-2=(x+2)(x-1)=0$, if $x=-2$ or $x=1$. The numerator is not equal to zero at these points, and so $x=-2$ and $x=1$ are vertical asymptotes.

23. $\lim_{t\to\infty}\left[2+\frac{5}{(t-2)^2}\right]=2$, and so $y=2$ is a horizontal asymptote. Next observe that

$\lim_{t\to 2^+} g(t)=\lim_{t\to 2^-}\left[2+\frac{5}{(t-2)^2}\right]=\infty$, and so $t=2$ is a vertical asymptote.

24. $\lim_{x\to\infty}\left[1+\frac{2}{x-3}\right]=1$ and $\lim_{x\to-\infty}\left[1+\frac{2}{x-3}\right]=1$, so $y=1$ is a horizontal asymptote.

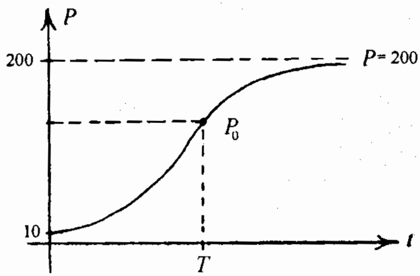
Next, we write $f(x)=1+\frac{2}{x-3}=\frac{x-3+2}{x-3}=\frac{x-1}{x-3}$, and observe that the denominator of $f(x)$ is equal to zero at $x=3$. However, since the numerator of $f(x)$ is not equal to zero at $x=3$, we see that $x=3$ is a vertical asymptote.

25. $\lim_{x\to\infty}\frac{x^2-2}{x^2-4}=\lim_{x\to\infty}\frac{1-\frac{2}{x^2}}{1-\frac{4}{x^2}}=1$ and so $y=1$ is a horizontal asymptote. Next, observe that the denominator $x^2-4=(x+2)(x-2)=0$ if $x=-2$ or 2. Since the numerator x^2-2 is not equal to zero at these points, the lines $x=-2$ and $x=2$ are vertical asymptotes.

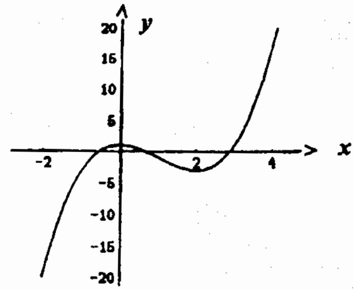
27. $g(x)=\dfrac{x^3-x}{x(x+1)}$; Rewrite $g(x)$ as $g(x)=\dfrac{x^2-1}{x+1}$ $(x\neq 0)$ and note that

$\lim_{x\to-\infty} g(x)=\lim_{x\to-\infty}\dfrac{x-\frac{1}{x}}{1+\frac{1}{x}}=-\infty$ and $\lim_{x\to\infty} g(x)=\infty$. Therefore, there are no horizontal asymptotes. Next, note that the denominator of $g(x)$ is equal to zero at $x=0$ and $x=-1$. However, since the numerator of $g(x)$ is also equal to zero when $x=0$, we see that $x=0$ is not a vertical asymptote. Also, the numerator of $g(x)$ is equal to zero when $x=-1$, so $x=-1$ is not a vertical asymptote.

32.



33.

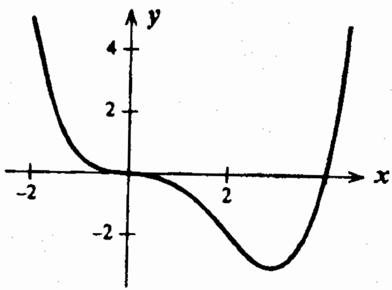


a. f is increasing on $(0,\infty)$. b. Yes, $P = 200$

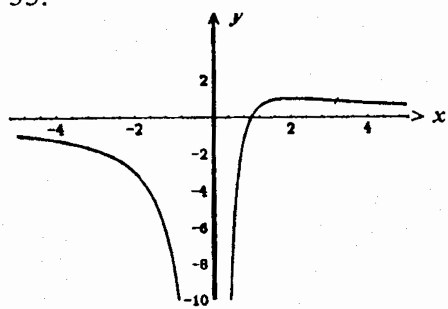
c. Concave up on $(0,T)$ and concave down on (T,∞).

d. Yes; at P_0. $P(t)$ is increasing fastest at $t = T$.

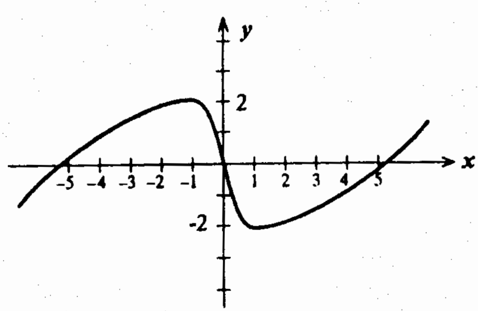
34.



35.



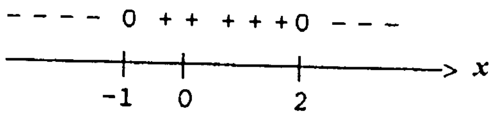
36.



41. $f(x) = -2x^3 + 3x^2 + 12x + 2$

We first gather the following information on the graph of f.

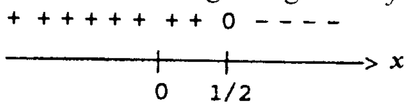
1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 2 as the y-intercept.
3. $\lim_{x\to-\infty}(-2x^3 + 3x^2 + 12x + 2) = \infty$ and $\lim_{x\to\infty}(-2x^3 + 3x^2 + 12x + 2) = -\infty$
4. There are no asymptotes because $f(x)$ is a polynomial function.
5. $f'(x) = -6x^2 + 6x + 12 = -6(x^2 - x - 2) = -6(x-2)(x+1) = 0$ if $x = -1$ or $x = 2$, the critical points of f. From the sign diagram



we see that f is decreasing on $(-\infty,-1) \cup (2, \infty)$ and increasing on $(-1,2)$.

6. The results of (5) show that $(-1,-5)$ is a relative minimum and $(2,22)$ is a relative maximum.

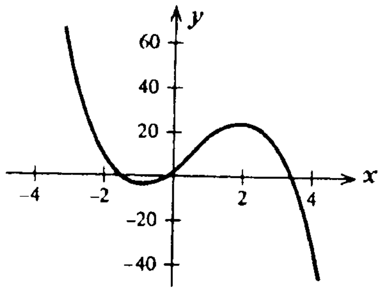
7. $f''(x) = -12x + 6 = 0$ if $x = 1/2$. The sign diagram of f''



shows that the graph of f is concave upward on $(-\infty,1/2)$ and concave downward on $(1/2, \infty)$.

8. The results of (7) show that $(\frac{1}{2}, \frac{17}{2})$ is an inflection point.

The graph of f follows.

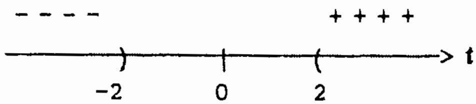


45. $f(t) = \sqrt{t^2 - 4}$.

We first gather the following information on f.

1. The domain of f is found by solving $t^2 - 4 \geq 0$ giving it as $(-\infty,-2] \cup [2,\infty)$.
2. Since $t \neq 0$, there is no y-intercept. Next, setting $y = f(t) = 0$ gives the t-intercepts as -2 and 2.
3. $\lim_{t \to -\infty} f(t) = \lim_{t \to \infty} f(t) = \infty$
4. There are no asymptotes.
5. $f'(t) = \frac{1}{2}(t^2 - 4)^{-1/2}(2t) = t(t^2 - 4)^{-1/2} = \frac{t}{\sqrt{t^2 - 4}}$.

Setting $f'(t) = 0$ gives $t = 0$. But $t = 0$ is not in the domain of f and so there are no critical points. The sign diagram for f' is



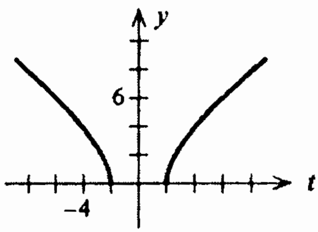
We see that f is increasing on $(2,\infty)$ and decreasing on $(-\infty,-2)$.

6. From the results of (5) we see that there are no relative extrema.
7. $f''(t) = (t^2 - 4)^{-1/2} + t(-\tfrac{1}{2})(t^2 - 4)^{-3/2}(2t) = (t^2 - 4)^{-3/2}(t^2 - 4 - t^2)$

$$= -\frac{4}{(t^2 - 4)^{3/2}}.$$

8. Since $f''(t) < 0$ for all t in the domain of f, we see that f is concave downward

everywhere. From the results of (7), we see that there are no inflection points. The graph of f follows.



53. $f(t) = \dfrac{t^2}{1+t^2}$.

We first gather the following information on the graph of f.

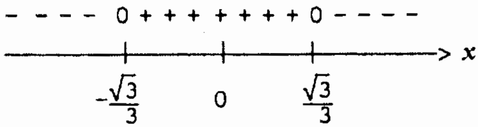
1. The domain of f is $(-\infty, \infty)$.
2. Setting $t = 0$ gives the y-intercept as 0. Similarly, setting $y = 0$ gives the t-intercept as 0.
3. $\lim\limits_{t\to-\infty} \dfrac{t^2}{1+t^2} = \lim\limits_{t\to\infty} \dfrac{t^2}{1+t^2} = 1$.
4. The results of (3) show that $y = 1$ is a horizontal asymptote. There are no vertical asymptotes since the denominator is not equal to zero.
5. $f'(t) = \dfrac{(1+t^2)(2t) - t^2(2t)}{(1+t^2)^2} = \dfrac{2t}{(1+t^2)^2} = 0$, if $t = 0$, the only critical point of f.

Since $f'(t) < 0$ if $t < 0$ and $f'(t) > 0$ if $t > 0$, we see that f is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.

6. The results of (5) show that (0,0) is a relative minimum.
7. $f''(t) = \dfrac{(1+t^2)^2(2) - 2t(2)(1+t^2)(2t)}{(1+t^2)^4} = \dfrac{2(1+t^2)[(1+t^2) - 4t^2]}{(1+t^2)^4}$

$$= \frac{2(1-3t^2)}{(1+t^2)^3} = 0 \text{ if } t = \pm\frac{\sqrt{3}}{3}.$$

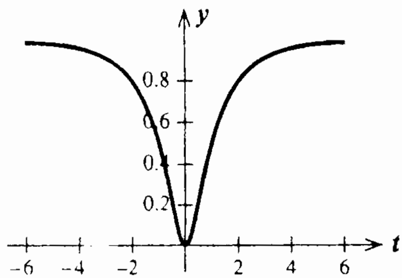
The sign diagram of f'' is



and shows that f is concave downward on $(-\infty,-\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3},\infty)$ and concave upward on $(-\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3})$.

8. The results of (7) show that $(-\frac{\sqrt{3}}{3},\frac{1}{4})$ and $(\frac{\sqrt{3}}{3},\frac{1}{4})$ are inflection points.

The graph of f follows.



63. a. Since $\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \frac{0.2t}{t^2+1} = \lim_{t\to\infty} \left[\frac{0.2}{t+\frac{1}{t^2}}\right] = 0$, $y = 0$ is a horizontal asymptote.

b. Our results reveal that as time passes, the concentration of the drug decreases and approaches zero.

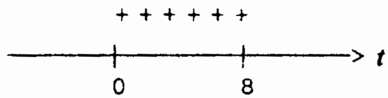
65. $G(t) = -0.2t^3 + 2.4t^2 + 60.$

We first gather the following information on the graph of G.

1. The domain of G is $(0,\infty)$.
2. Setting $t = 0$ gives 60 as the y-intercept.

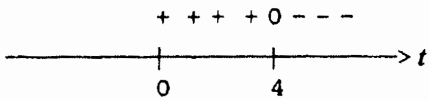
Note that Step 3 is not necessary in this case because of the restricted domain.

4. There are no asymptotes since G is a polynomial function.
5. $G'(t) = -0.6t^2 + 4.8t = -0.6t(t - 8) = 0$, if $t = 0$ or $t = 8$. But these points do not lie in the interval $(0,8)$, so they are not critical points. The sign diagram of G'



shows that G is increasing on $(0,8)$.

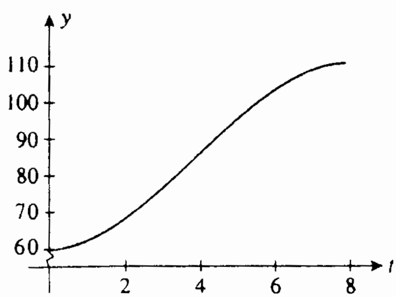
6. The results of (5) tell us that there are no relative extrema.
7. $G''(t) = -1.2t + 4.8 = -1.2(t - 4)$. The sign diagram of G'' is



and shows that G is concave upward on $(0,4)$ and concave downward on $(4,8)$.

8. The results of (7) shows that $(4,85.6)$ is an inflection point.

The graph of G follows.



69. $T(x) = \dfrac{120x^2}{x^2 + 4}$.

We first gather the following information on the function T.

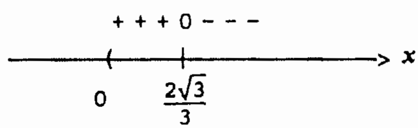
1. The domain of T is $[0,\infty)$.
2. Setting $x = 0$ gives 0 as the y-intercept.
3. $\lim\limits_{x\to\infty} \dfrac{120x^2}{x^2+4} = 120.$
4. The results of (3) show that $y = 120$ is a horizontal asymptote.
5. $T'(x) = 120\left[\dfrac{(x^2+4)2x - x^2(2x)}{(x^2+4)^2}\right] = \dfrac{960x}{(x^2+4)^2}$. Since $T'(x) > 0$

if $x > 0$, we see that T is increasing on $(0,\infty)$.

6. There are no relative extrema in $(0,\infty)$.
7. $T''(x) = 960\left[\dfrac{(x^2+4)^2 - x(2)(x^2+4)(2x)}{(x^2+4)^4}\right]$

$$= \frac{960(x^2+4)[(x^2+4)-4x^2]}{(x^2+4)^4} = \frac{960(4-3x^2)}{(x^2+4)^3}.$$

The sign diagram for T'' is



We see that T is concave downward on $(\frac{2\sqrt{3}}{3},\infty)$ and concave upward on $(0, \frac{2\sqrt{3}}{3})$.

8. We see from the results of (7) that $(\frac{2\sqrt{3}}{3}, 30)$ is an inflection point.

The graph of T follows.