MATH180 – HOMEWORK SOLUTIONS

HOMEWORK #1

Section 2.1: Problems 5, 11, 15, 19, 33, 39, 41, 47, 59, 61, 69, 71, 75

Section 2.2: Problems 7, 17, 27, 33, 45, 47, 51, 55, 57, 59, 61, 65

Section 2.3: Problems 5, 11, 13, 17, 21, 29, 37, 39, 43, 51, 65

Section 2.4: Problems 3, 5, 11, 15, 19, 29, 51, 55, 59, 77, 79

Section 2.5: Problems 3, 5, 7, 11, 13, 15, 17, 19, 29, 39, 47, 57, 59, 65

Krzysztof Galicki

Problem 2.1.5 If $f(x) = 2x + 5$

$$f(-a) = 2(-a) + 5 = -2a + 5,$$

$$f(a + h) = 2(a + h) + 5 = 2a + 2h + 5,$$

$$f(a^2) = 2a^2 + 5,$$

$$f(a - 2h) = 2(a - 2h) + 5 = 2a - 4h + 5,$$

$$f(2a - h) = 2(2a - h) + 5 = 4a - 2h + 5.$$

Problem 2.1.11 We have

$$f(-2) = (-2)^2 + 1 = 4 + 1 = 5,$$

$$f(0) = 1,$$

$$f(1) = \sqrt{1} = 1.$$

Problem 2.1.15

(a) $f(0) = -2$,

(b) $f(2) = 3$ and $f(1) = 0$,

(c) the domain of $f(x)$ is the closed interval $[0,6]$,

(d) the range of $f(x)$ is the closed interval $[-2,6]$,

Problem 2.1.19 The point $P=(-2,-3)$ does lie on the graph of

$$f(t)=\frac{|t-1|}{t+1}$$

because $f(-2)=|-2-1|/(-1)=-3$.

Problem 2.1.33 If

$$f(x)=\frac{\sqrt{1-x}}{x^2-4}$$

then we must have

$$1-x\geq 0, \qquad \text{and} \qquad x^2-4\neq 0.$$

Hence, $x\leq 1$ and $x\neq\pm 2$. We can write this as a union of two intervals $(-\infty,-2)\cup(-2,1]$.

Problem 2.1.39 $f(x)=2+\sqrt{x}$. The domain of $f(x)$ is $[0,\infty)$. The range of $f(x)$ is $[2,\infty)$. And the graph is given below

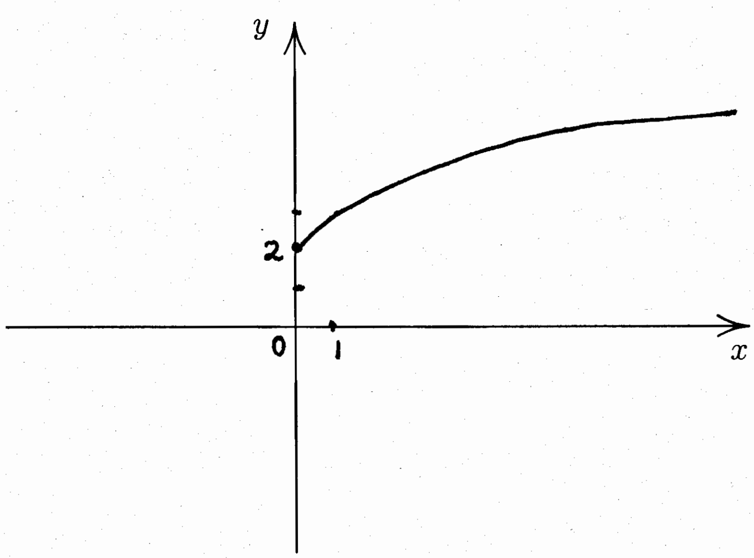

Fig. The graph of $f(x)$

Problem 2.1.41 $f(x)=\sqrt{1-x}$. The domain of $f(x)$ is $(-\infty,1]$. The range of $f(x)$ is $[0,+\infty)$. And the graph is given below

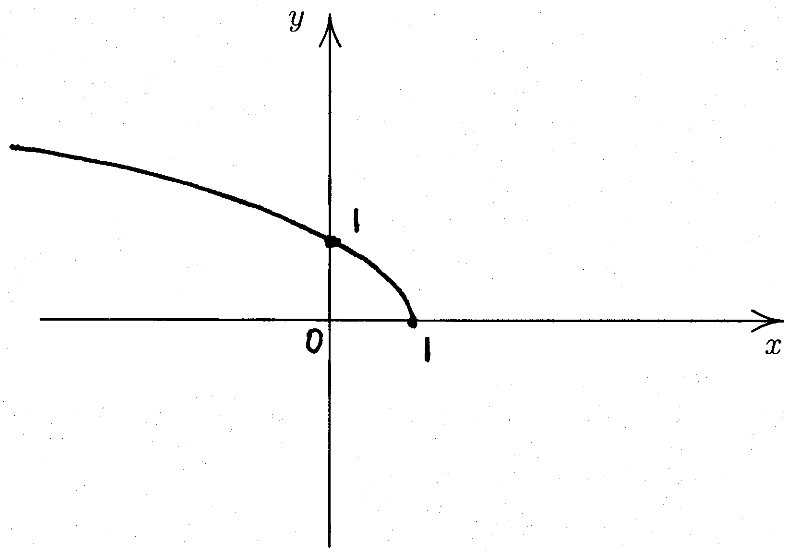

Fig. The graph of $f(x)$.

Problem 2.1.47

$$f(x) = \begin{cases} -x+1 & \text{if } x \leq 1 \\ x^2 - 1 & \text{if } x > 1. \end{cases}$$

The domain of $f(x)$ is all real numbers $(-\infty, +\infty)$ and the range is $[0, +\infty)$ (note that $-x+1$ is positive for $x \leq 1$ and $x^2 - 1$ is also positive for $x > 1$.)

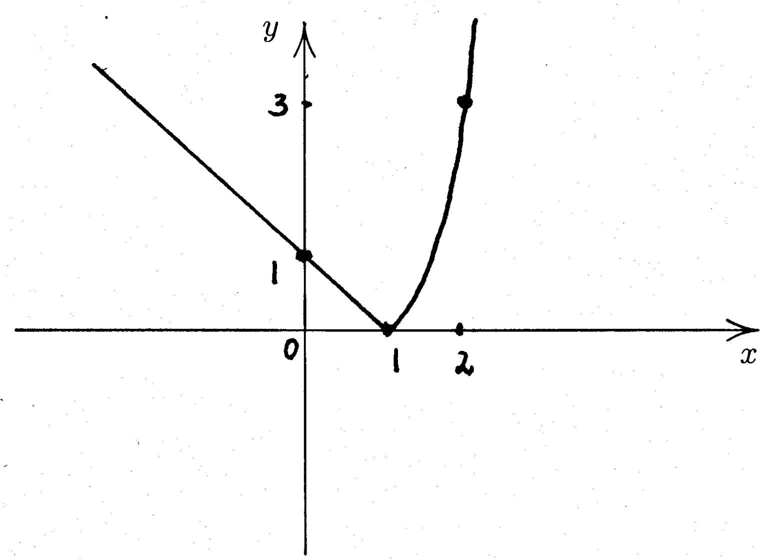

Fig. The graph of $f(x)$.

Problem 2.1.59 Since $V(r) = \frac{4}{3}\pi r^3$ we get

$$V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi 2^3(r)^3 = 8\frac{4}{3}\pi r^3 = 8V(r).$$

Hence, a ball twice the radius of a given one has the volume which is 8 times larger.

Problem 2.1.61 Before 1990 the sales of cassettes were greater. After 1990 the sales of CDs were greater. In the year 1990 the sales of CDs and cassettes were equal and they were about 3.5 billion dollars each.

Problem 2.1.69 Assume in each case we rent a truck for a day and drive x miles that day. The rental cost with company A (Ace) is

$$A(x) = 30 + .45x$$

and the rental cost with company B (Acme) is

$$B(x) = 25 + .5x$$

Let us plot the two functions. Note that the costs are equal at $30 + .45x = 25 + .5x$ or $x = 100$. So, if you drive 100 miles the first day, it does not matter which company you choose.

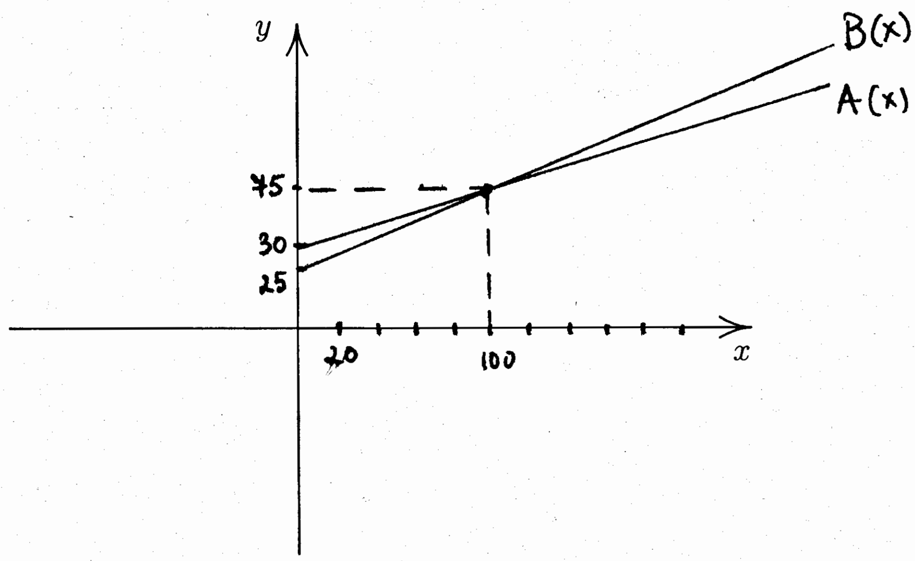

Fig. The graph of $A(x)$ and $B(x)$

Since $A(70) = 30{+}.45(70) = 30{+}31.5 = 61.50$ and $B(70) = 25{+}.5(70) = 35{+}25 = 60$ one should choose company B (Acme). **Beware: The answers given in your textbook ((b) and (c)) are wrong!**

Problem 2.1.71 Since in 1986 the building is worth 1 million dollars and 50 years later it is worth nothing, assuming linear depreciation we see that the value depreciates at the rate 20,000 dollars per year. Assume that $t = 0$ corresponds to year 1986. Then we can write the value of the building as a function of time

$$V(n) = 1,000,000 - 20,000n.$$

Now, 2001 corresponds to $n = 15$, 2005 to $n = 19$ and 2009 to $n = 23$. The value of the building in these years will be $V(15) = 700,000$, $V(19) = 620,000$, $V(23) = 540,000$.

Problem 2.1.75 The worker's efficiency function in the first 4 hours of work ($0 \le t \le 4$) is

$$N(t) = -t^3 + 6t^2 + 15t.$$

By 9:00 a.m. an average worker will make $N(1) = -1 + 6 + 15 = 20$ walkie-talkies. By 10:00 a.m. an average worker will make $N(2) = -8 + 24 + 30 = 46$ units. Hence, between 9:00 and 10:00 a.m. an average worker will make 26 walkie-talkies.

Problem 2.2.7 Let $f(x) = x^3 + 5, g(x) = x^2 - 2$ and $h(x) = 2x + 4$. Then

$$\frac{fg}{h}(x) = \frac{(x^3+5)(x^2-2)}{2x+4} = \frac{(x^5 - 2x^3 + 5x^2 - 10)}{2x+4}.$$

Problem 2.2.17 Let $f(x) = x - 1, g(x) = \sqrt{x+1}$ and $h(x) = 2x^3 - 1$. Then

$$\frac{f-h}{g}(x) = \frac{x - 1 - (2x^3 - 1)}{\sqrt{x+1}} = \frac{x - 2x^3}{\sqrt{x+1}}.$$

Problem 2.2.27 Let $f(x) = \sqrt{x} + 1$ and $g(x) = x^2 - 1$. Then

$$f \circ g(x) = f(g(x) = f(x^2 - 1) = \sqrt{x^2 - 1} + 1$$

and

$$g \circ f(x) = g(f(x) = g(\sqrt{x} + 1) = (\sqrt{x} + 1)^2 - 1 = x + 2\sqrt{x}.$$

Problem 2.2.33 Let $f(x) = \frac{1}{2x+1}$ and $g(x) = \sqrt{x}$. If $h(x) = g \circ f(x) = g(f(x)$ then

$$h(2) = g(f(2)) = g(\frac{1}{5}) = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

Problem 2.2.45 Let $f(x) = 4 - x^2$. We have

$$f(a+h) - f(a) = 4 - (a+h)^2 - [4 - a^2] = 4 - a^2 - 2ah - h^2 - 4 + a^2 = -2ah - h^2 = -h(h+2a).$$

Problem 2.2.47 Let $f(x) = x^2 + 1$. We have

$$\frac{f(a+h) - f(a)}{h} = \frac{(a+h)^2 + 1 - [a^2 + 1]}{h} = \frac{a^2 + 2ah + h^2 + 1 - a^2 - 1}{h} =$$

$$= \frac{2ah + h^2}{h} = 2a + h.$$

Problem 2.2.51 Let $f(x) = \frac{1}{x}$. We have

$$\frac{f(a+h) - f(a)}{h} = \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \frac{\frac{a-(a+h)}{a(a+h)}}{h} = \frac{\frac{-h}{a(a+h)}}{h} = -\frac{1}{a(a+h)}.$$

Problem 2.2.55 The product of the function of number of shares times the function of the price is **the value in dollars of nancy's shares of IBM at a time** t.

Problem 2.2.57 The carbon monoxide pollution in parts per million at time t.

Problem 2.2.59 The cost function is $C(x) = 12,000 + .6x$.

Problem 2.2.61 The total cost function from Problem 60 is

$$C(x) = C_0 + V(x) = 0.000003x^3 - 0.03x^2 + 200x + 100,000.$$

If the revenue function in dollars is

$$R(x) = -0.1x^2 + 500x$$

then the profit function is

$$P(x) = R(x) - C(x) = -0.1x^2 + 500x - (0.000003x^3 - 0.03x^2 + 200x + 100,000) =$$

$$= -0.000003x^3 - 0.07x^2 + 300x - 100,000.$$

In particular,

$$P(1500) = 182,375.$$

Problem 2.2.65 The occupancy rate is

$$r(t) = \frac{10}{81}t^3 - \frac{10}{3}t^2 + \frac{200}{9}t + 55$$

and the monthly revenue function given occupancy rate r is

$$R(r) = -\frac{3}{5000}r^3 + \frac{9}{50}r^2.$$

Let us compute occupancy rate at the beginning of January ($t = 0$) and beginning of June ($t = 5$):

$$r(0) = 55\%,$$

$$r(5) = \frac{10}{81}125 - \frac{10}{3}25 + \frac{200}{9}5 + 55 = \frac{1,125}{81} - \frac{250}{3} + \frac{1000}{9} + 55 = 98.2\%.$$

At these two rates the monthly revenue is

$$R(55) = -\frac{3}{5000}(55)^3 + \frac{9}{50}(55)^2 = 444,700,$$

$$R(98.2) = -\frac{3}{5000}(98.2)^3 + \frac{9}{50}(98.2)^2 = 1,167,600.$$

Problem 2.3.5 Here y **is** a linear function of x and $y = \frac{2x+9}{4} = \frac{1}{2}x + \frac{9}{4}$.

Problem 2.3.11 Here $G(x)$ is a polynomial of degree 6. More precisely,

$$G(x) = 2(x^6 - 9x^4 + 27x^2 - 27) = 2x^6 - 19x^4 + 54x^2 - 54.$$

Problem 2.3.13 Here $f(t)$ is a sum of a polynomial ($2t^3$) and a power function ($3\sqrt{t}$). Hence, $f(t)$ is not a polynomial and it is not a rational function.

Problem 2.3.17 The cost function is

$$C(x) = 40,000 + 8x.$$

The revenue function is $R(x) = px = 12x$ and, hence, the profit function

$$P(x) = R(x) - C(x) = 12x - (8x + 40,000) = 4x - 40,000.$$

Now, $P(8,000) = -8,000$, $P(12,000) = +8,000$. $x = 10,000$ is the "break-even" point when $P(10,000) = 0$.

Problem 2.3.21 The dosage is

$$D(4) = \frac{4+1}{24}500 = \frac{2,500}{24} = 104.16 \text{ mg}.$$

Problem 2.3.29 In this problem we have to write down the equation of a line in (T, N) plane (N is the number of chirps per minute and T the air temperature). The problem specifies two points on this line: $(70, 120)$ and $(80, 160)$. Hence, the equation of the line is

$$N(T) - 120 = \frac{160 - 120}{80 - 70}(T - 70),$$

or

$$N(T) - 120 = 4(T - 70),$$

or

$$N(T) = 4T - 160.$$

In particular, $N(102) = 408 - 160 = 248$. Of course, we can also write T as a function of N, namely $T = \frac{N+160}{4} = \frac{1}{4}N + 40$.

Problem 2.3.37 For $0 \leq t \leq 2$ we have

$$f(t) = \frac{110}{\frac{1}{2}t + 1}, \qquad g(t) = 26(t^2/4 - 1)^2 + 52,$$

$$h(t) = f(t) - g(t) = \frac{110}{\frac{1}{2}t + 1} - 26(t^2/4 - 1)^2 - 52.$$

Hence,

$$f(0) = 110, g(0) = 78, \qquad f(0) - g(0) = 32$$
$$f(1) = 73.33, g(1) = 66.63, \qquad f(0) - g(0) = 6.71$$
$$f(2) = 55, g(2) = 52, \qquad f(0) - g(0) = 3.$$

We conclude that the difference in price between OEM and non-OEM decreases by 29 dollars.

Problem 2.3.39 Let $p(x) = -x^2 + 16$. We plot the demand curve

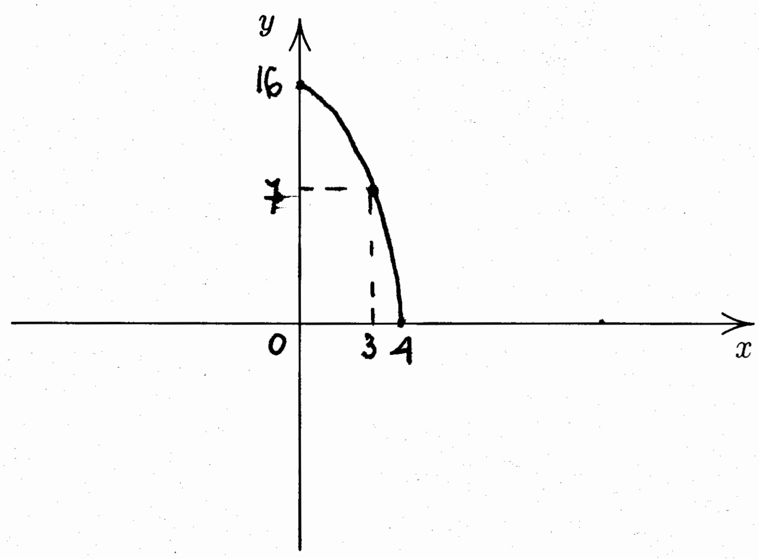

Fig. The graph of the demand curve $p(x) = -x^2 + 16$.

When $p = 7$ we have $7 = -x^2 + 16$, or $x^2 = 9$. Since x is positive $x = 3$ (that is, 3000 units).

Problem 2.3.43 Let $p(x) = x^2 + 16x + 40$. We plot the supply curve

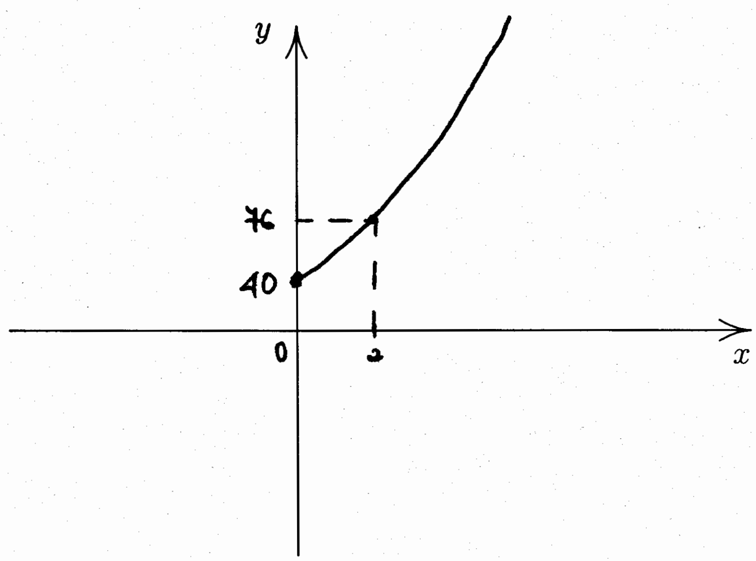

Fig. The graph of the supply curve $p(x) = x^2 + 16x + 40$.

At the supply level of 2000 units $x = 2$ and the price $p(2) = 2^2 + 32 + 40 = 76$.

Problem 2.3.51 We know that the supply function has the form

$$p(x) = a\sqrt{x} + b.$$

We also know that

$$p(10,000) = a\sqrt{10,000} + b = 100a + b = 20$$

and

$$p(62,500) = a\sqrt{62,500} + b = 250a + b = 35.$$

Subtracting the first equation from the second gives $150a = 15$, hence $a = \frac{1}{10}$. Then using the first equation, we get

$$10 + b = 20,$$

which means that $b = 10$. We have

$$p(x) = \frac{1}{10}\sqrt{x} + 10.$$

In particular, $p(40,000) = \frac{200}{10} + 10 = 30$ dollars.

Problem 2.3.65 If x is the number of trees beyond 22 trees/acre, then the tree density is $x + 22$. However, the average yield is now $36 - 2x$ bushels/acre. The desired function is simply the product

$$f(x) = (x + 22)(36 - 2x).$$

Its domain, as stated by the problem is $x \in [0, 18]$. $x = 0$ corresponds to the original situation, while $x = 18$ gives zero yield. $f(x)$ cannot be negative. Also, we do not have any information what happens to the crop when one decreases tree density (negative x).

Problem 2.4.3 Here the limit at $x = 3$ is equal to 3 and the value of the function $f(3) = 4$.

Problem 2.4.5 Here the limit at $x = -2$ is equal to 3 and the value of the function $f(-2) = 2$.

Problem 2.4.11 Let $f(x) = |x|/x$. We have

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-1	-1	-1	1	1	1

The limit at $x = 0$ does not exists. The left limit equals -1 but the right limit equals +1.

Problem 2.4.15 Let $f(x) = \frac{x^2+x-2}{x-1} = \frac{(x+2)(x-1)}{x-1}$. We have

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	2.9	2.99	2.999	3.001	3.01	3.1

The limit at $x = 1$ exists and equals 3.

Problem 2.4.19 Let

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$$

The limit at $x = 1$ exists and it is equal to 1. See the graph below.

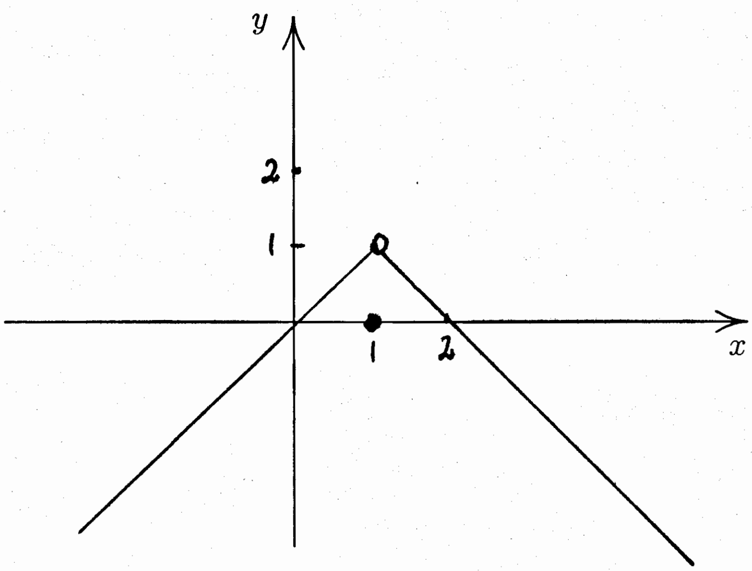

Fig. The graph of $f(x)$.

Problem 2.4.29

$$\lim_{x\to 1}(2x^3 - 3x^2 + x + 2) = 2 - 3 + 1 + 2 = 2.$$

Problem 2.4.51

$$\lim_{x\to 0}\frac{x^2 - x}{x} = \lim_{x\to 0}(x - 1) = -1.$$

Problem 2.4.55

$$\lim_{x\to 1}\frac{x}{x-1} \quad \text{does not exist.}$$

Problem 2.4.59

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}=\lim_{x\to 1}\frac{1}{\sqrt{x}+1}=\frac{1}{2}$$

Problem 2.4.77

$$\lim_{x\to -\infty}\frac{x^4+1}{x^3-1}=-\infty.$$

Problem 2.4.79

$$\lim_{x\to \infty}\frac{x^5-x^3+x-1}{x^6+2x^2+1}=0.$$

Problem 2.5.3

$$\lim_{x\to -1^-} f(x)=+\infty, \qquad \lim_{x\to -1^+} f(x)=2.$$

Hence,

$$\lim_{x\to -1} f(x)= \text{ does not exist.}$$

Problem 2.5.5

$$\lim_{x\to 1^-} f(x)=0, \qquad \lim_{x\to 1^+} f(x)=2.$$

Hence,

$$\lim_{x\to 1} f(x)= \text{ does not exist.}$$

Problem 2.5.7

$$\lim_{x\to 0^-} f(x)=-2, \qquad \lim_{x\to 0^+} f(x)=2.$$

Hence,

$$\lim_{x\to 0} f(x)= \text{ does not exist.}$$

Problem 2.5.11 TRUE.

Problem 2.5.13 FALSE. The limit exists and equals 3.

Problem 2.5.15 TRUE.

Problem 2.5.17 FALSE. The limit exists but is is equal to 2. The value of the function $f(2) = 1$.

Problem 2.5.19 TRUE. The right limit at $x = 4$ is $+\infty$.

Problem 2.5.29

$$\lim_{x\to 0^+} \sqrt{x} = 0.$$

Problem 2.5.39 Let

$$f(x) = \begin{cases} 2x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

We have:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 0.$$

Problem 2.5.47 The function

$$f(x) = \begin{cases} x+5 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ -x^2+5 & \text{if } x > 0 \end{cases}$$

is discontinuous only at $x = 0$. We have

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 5,$$

however,

$$2 = f(0) \neq \lim_{x\to 0} f(x).$$

Problem 2.5.57 Let

$$f(x) = \frac{2x+1}{x^2+x-2} = \frac{2x+1}{(x-1)(x+2)}.$$

The function is not defined for $x = 1$ and $x = -2$ and it is also discontinuous exactly at these two points. One cannot define $f(1)$ and $f(2)$ to make $f(x)$ continuous at these two points.

Problem 2.5.59 The function

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$$

is continuous everywhere.

Problem 2.5.65 The function $f(x) = |x + 1|$ is continuous everywhere.