MATH 180 INSTRUCTORS

The attached pages are the review material for the final exam. This includes a list of topics and two problem sets from recent final exams. As in past exams, the students are allowed to bring a 5 x 8 card with notes on both sides into the exam. They may use a graphing calculator as an aid or to check a result, but all problems are to be solved analytically—i.e., without the use of the calculator. The students must show all of their work.

The review topics and the sets of problems will be available to the students from the eReserve by downloading (use password lobo180 and request the package titled MATH 180 FINAL EXAM REVIEW). This package will also be available for checkout (2 hrs.) at the Reserve desk at Zimmermann, where the students can make hardcopies. The package will also be available on the Math webpage for downloading. All material should be available by Monday 4/25.

I am also putting together a review package for the 3rd exam, and that should also be available on Monday from the sources listed above. Note cards are **not** allowed for the third exam.

Please contact me if you have any problems with the review material or any suggestions for changes.

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MATH 180 REVIEW TOPICS FOR FINAL EXAM SPRING 05

The following list of topics will be covered on the Final Exam. Two sets of review problems are given. These problems were used in recent final exams. Students may use a 5" x 8" card (both sides) with notes during the exam.

1. Derivatives: there will be 4 or 5 derivatives that test their knowledge of the differentiation rules: product, quotient, and chain rules, including the exponential function, the logarithm function, and the basic power function x^n. The functions in the two sample exams are a good sample of what the students can expect. There will not be a problem involving the limit definition of the derivative.
2. Integrals: there will be 4 or 5 integrals to evaluate, some of which will be definite integrals, and some of which will require the substitution method. The two sample exams have several drill problems for this topic. Review also integrals of the form $\int \frac{x}{\sqrt{x+1}}dx$.
3. Basic applications of the derivative and integral: Given a function f(x), find the equation of the tangent line through a given point and find the area under the curve
4. Given the *graph* of a function f(x), answer questions about the first and second derivatives of f(x). Students will be required to explain their answers briefly.
5. Given the graph of *the derivative* $g'(x)$, answer questions about g(x) and $g''(x)$. Explain your answer.
6. Given f(x) as an expression in x, determine intervals of increasing and decreasing f(x), locations of local max/min points, intervals for positive and negative concavity, locations of inflection points.
7. Business-related problems: given information concerning revenue and cost functions, determine the profit function and the production levels for maximum profit. Also, given the *marginal* revenue, cost, and/or profit, find the corresponding cost/revenue/profit functions by integration.
8. Optimization problems related to designs of rectangular containers or of rectangular farming/gardening areas. Problems will ask for designs that maximize volume, minimize or maximize an area, or minimize cost. Refer to problems 1-3 and 5-8 on pages 325-326 and problems 40 and 42 on page 331.
9. Problems involving exponential growth or decay of the form $P = P_0e^{kt}$. These could be associated with an investment problem, growth of a bacteria population, or radioactive decay.
10. Analysis of functions that are piecewise defined.

Set 1

1. Write the first derivative for each of the following functions. DO NOT SIMPLIFY.
   (a) $y = -\dfrac{3x^5}{7} + \dfrac{1}{\sqrt{3x}} - \sqrt[3]{x} + 3$
   (b) $y = x\ln(x + \dfrac{1}{x})$
   (c) $y = (x^3 - 1)^{10}(2 - 3x)^9$
   (d) $y = \dfrac{e^{2x}}{\ln(x+1)}$
   (e) $y = [\ln\sqrt{2-3x}]^{\frac{1}{2}}$

2. Given the function $f(x) = 3 - x^3 - e^{-x}$,
   (a) Write the equation of the tangent line to the graph of f(x) at x = 0
   (b) Determine the area under the graph of f(x) on the interval $0 \le x \le 1$.

3. A profit function for a certain business is given by $P(x) = \dfrac{x^3}{3} - \dfrac{5x^2}{2} + 4x + 3$,
   where x is the number of items sold, in millions, and P is the profit in millions of dollars.
   (a) Determine the production levels (the values of x) for which the profit P(x) is increasing and decreasing.
   (b) Determine the production levels over which the graph is concave up and concave down.
   (c) Determine the values of x at which the maximum and minimum values of the function occur.

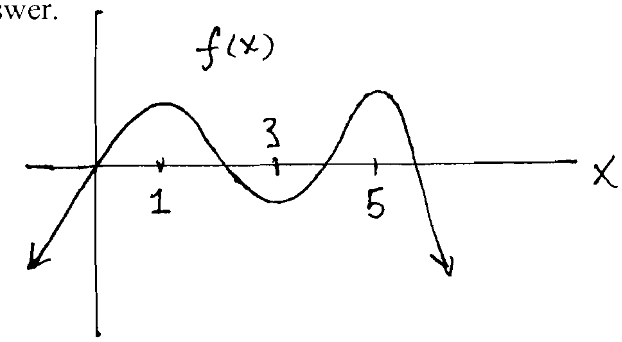
4. An open rectangular box with a square base is to have a volume of 32 cubic feet. Determine the dimensions of the box so that the surface area is minimized.

5. Evaluate the following definite or indefinite integrals.
   (a) $\int \dfrac{x^2+1}{x}\,dx$
   (b) $\int_0^1 3e^{-3t}\,dt$
   (c) $\int xe^{-x^2}\,dx$
   (d) $\int (2x-1)(x^2 - x - 17)^3\,dx$

6. The demand equation for an item is $p = 400 - 4x$ and the cost function is given by $C(x) = 150 + 160x - 2x^2$, where $0 \le x \le 80$. Determine the value of x and the price per item that produces the maximum profit.

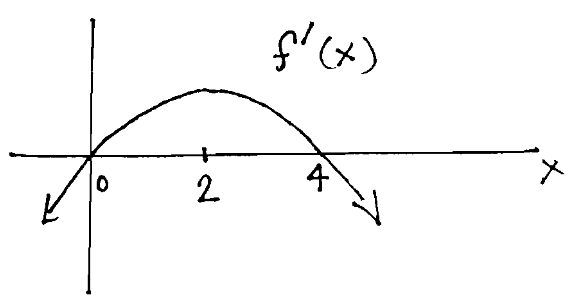
7. The population of a laboratory bacteria colony is observed to grow exponentially such that the population after one hour is 10 times the starting population.
   a. Determine the equation for the population P(t) if the initial population was 100 bacteria.
   b. How many hours will it take for the population to grow to 1,000,000 bacteria?
   c. What is the rate of growth of the population, in bacteria per hour, when the population is 10,000,000 bacteria?

8. The figure below is the graph of the function f(x). Answer the following questions concerning the first and second derivatives of f(x). You must explain each answer.



   a. Over what intervals is the first derivative positive? Explain.
   b. Over what intervals is the second derivative negative? Explain.
   c. At what points, if any, is the second derivative equal to zero? Explain.

9. The figure shows the **first derivative** of a function, or $f'(x)$.
(a) Over what interval in x is the function f(x) increasing? Explain.
(b) For what value of x does the function have a local maximum? Explain.
(c) Over what interval in x is the function concave down? Explain.



Set 2

1. (28 pts.) Determine the first derivative of each of the following functions. DO NOT SIMPLIFY.

(a) $y = \frac{3}{2x} - 5x^3 + \frac{2}{\sqrt{x}}$

(b) $y = (3x^2 - 2x)^5$

(c) $y = e^x \ln(x^2 + 1)$

(d) $y = \frac{\ln(1-3x)}{e^{1-3x}}$

2. (28 pts.) Evaluate the following integrals.

(a) $\int (x^3 + 3x^2 - \frac{1}{2x^2})dx$

(b) $\int_0^1 \frac{e^{-3x}}{6} dx$

(c) $\int \frac{\ln x}{x} dx$

(d) $\int \frac{x^5 + 1}{x^2} dx$

3. (24 pts.) Given the function $f(x) = 5 - x^2 + 5e^{x-1}$:

(a) Find the equation of the tangent line to f(x) at x = 1.

(b) Determine $\int f(x)dx$

(c) Determine the area under the graph of f(x) over the interval $0 \le x \le 1$.

4. (18 pts.) The population of a certain type of bacteria grows exponentially such that an initial population of 150 grows to a value of 450 after 5 hours.

(a) Determine the equation for P(t) , the population at any time t (in hours).

(b) Determine the population after 10 hours.

(c) How fast is the population increasing (in bacteria per hour) when the number of bacteria is 10,000 ?

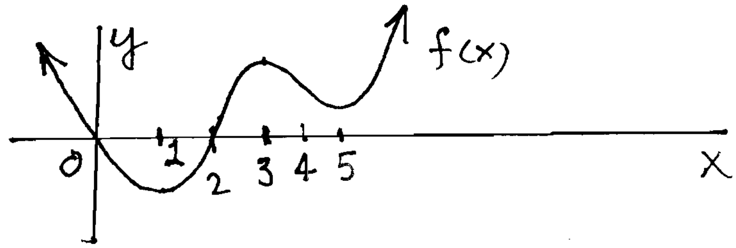
5. (24 pts.) Given the graph of the function f(x) below, answer the following questions about the first and second derivatives of f(x). Explain each answer briefly.

(a) List the intervals over which $f'(x)$ is positive. Explain.

(b) List the intervals over which $f''(x)$ is positive. Explain.

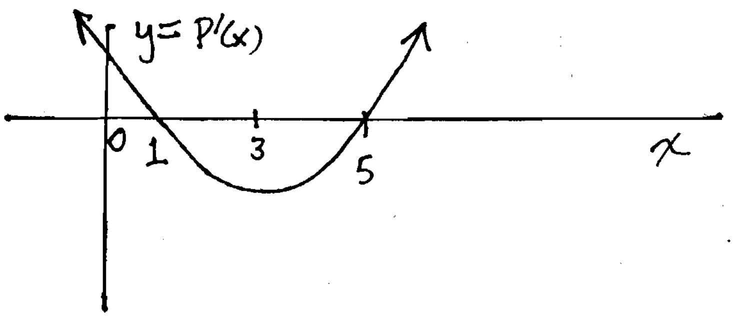
(c) List the values of x for which the first derivative is 0. Explain.

(d) Estimate the values of x for which the second derivative is 0. Explain.

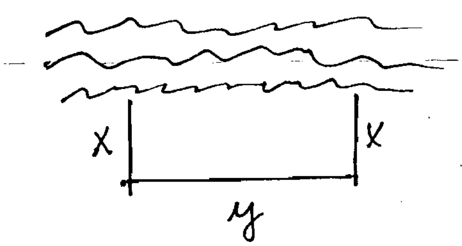


6. (24 pts.) A graph of the <u>marginal</u> profit, or $P'(x)$, for a business is shown below. Answer the following questions about the profit P(x) , where x is the number of items produced and sold.

(a) List the intervals in x for which the profit is decreasing. Explain.
(b) List the values of x for which the profit is at a local maximum point. Explain.
(c) List the values of x for which the profit function is concave up. Explain.



7. (18 pts.) A farmer plans to enclose a rectangular garden area next to a river. The area of the garden should be 5,000 square meters. The garden will be fenced on 3 sides but not along the river, as shown in the figure. Choose the dimensions of the garden so that the minimum amount (length) of fence material is required.



8. (18 pts.) Given the function $f(x) = \frac{\ln x}{x}$, x > 0 , find all critical points, identify any local maximum or minimum locations, and locate all inflection points if any.

9. (18 pts.) The marketing department of a business has determined that the demand for a product can be modeled by the demand function $p(x) = \frac{40}{\sqrt{x}}$, where p is the price per item and x is the number of items produced and sold. The cost of producing x items is given by the cost function C(x) = 2x + 50. Determine the value of x and the price that produces the maximum profit.