Problem 1.

a) $y(x) = x^5 + e^{5x} + x^e, \quad y'(x) = 5x^4 + 5^{5x} + ex^{e-1}.$

b) $y(x) = xe^x, \quad y'(x) = e^x + xe^x.$

c) $y(x) = \ln(x^2+1), \quad y'(x) = \frac{2x}{x^2+1}.$

d) $y(x) = e^{-2x^2}, \quad y'(x) = -4xe^{-2x^2}.$

e) $y(x) = \frac{x}{\ln x}, \quad y'(x) = \frac{\ln x - 1}{(\ln x)^2}.$

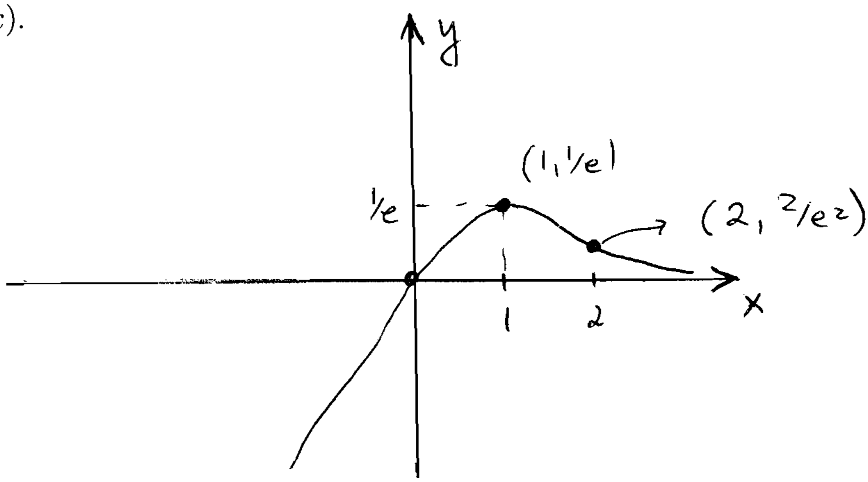
Problem 2.

a) $y'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$. As e^{-x} is never equal to zero, $y'(x) = 0$ only when $1 - x = 0$, or $x = 1$. By the first derivative test $y'(x) > 0$ for $x < 1$ ($y(x)$ increasing) and $y'(x) < 0$ for $x > 1$ ($y(x)$ decreasing) so that $(1, 1/e)$ must be a local minimum. It is also the absolute maximum as there are no other critical points.

b) $y''(x) = -e^{-x} - (1-x)e^{-x} = (-2+x)e^{-x}$. As in (a) $y''(x) > 0$ when $x \in (2, \infty)$ (concave up) and $y''(x) > 0$ when $x \in (-\infty, 2)$ (concave down). Hence $x = 2$ is the only inflection point. ($y(2) = 2/e^2$.)

c) The limit at minus infinity is clearly $-\infty$. The limit when $x \to \infty$ equals to 0.

d) Graph of $y(x)$.



Problem 3.

a) We have $P(t) = 200e^{kt}$. We know that $P(10) = 400 = 200e^{10k}$, and hence $k = \frac{1}{10}\ln 2 =\simeq 0.0693$.

b) $P(30) = 200e^{30k} = 253e^{2.08} = 1,600$ thousand.

c) $2,000 = 200e^{kT}$, hence $T = \frac{1}{k}\ln 10 = 33.23$. Hence, the population will reach 1 million in the year 2014.

Problem 4.

a) $\int (x^2 + 1/x)dx = \frac{1}{3}x^3 + \ln|x| + C.$

b) $\int_0^1 3e^{-2t}dx = -\frac{3}{2}e^{-2t}]_0^1 = \frac{-3}{2e^2} + \frac{3}{2}.$

c) $\int xe^{-x^2}dx = \int e^{-u}\frac{1}{2}du = -\frac{1}{2}e^{-u} + C = -\frac{1}{2}e^{-x^2} + C$, where $u = u(x) = x^2$.

d) $\int \frac{\ln x}{x} dx = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$, where $u = u(x) = \ln x$.

e) $\int_1^3 (x^2 - x^3)) dx = \frac{1}{3}x^3 - \frac{1}{4}x^4]_1^3 = (27/3 - 81/4) - (1/3 - 1/4) = 26/3 - 20 = -11\frac{1}{3}$.

Problem 5. With annual compounding the future value after 6 years will be

$$P(6) = 5000(1 + 0.05)^6 = 5000(1.05)^6 = 5000 \cdot 1.340 = 6700.48$$

With continuous compounding after 6 years we get

$$P(6) = 5000e^{.049 \cdot 6} = 5000e^{0.29} = 6708.92$$

Hence, the second option is slightly better.