MATHEMATICS 180

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TOTAL _____

Problem 1.(*30 pts.*) Let $f(x) = x^3 - 3x + 2$.

a) Compute the first derivative $f'(x)$ and find all critical points. Determine the intervals when the function is increasing and when it is decreasing. Use the first derivative test to identify local extrema.

b) Find the second derivative $f''(x)$, all inflection points and intervals where $f(x)$ is concave up and intervals where it is concave down.

c) Graph $f(x)$.

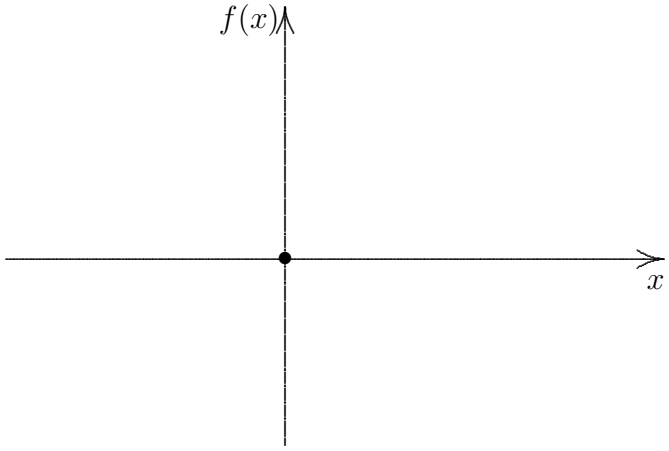

Fig. 1. The graph of $f(x) = x^3 - 3x + 2$

Problem 2. (*30 pts.*) Let $f(x) = 5x + \frac{35}{x}$.

a) Compute the first derivative $f'(x)$ and find all critical points. Determine the intervals when the function is increasing and when it is decreasing. Identify local minima and maxima using the first derivative test.

b) Find the second derivative $f''(x)$, all inflection points and intervals where $f(x)$ is concave up and intervals where it is concave down.

c) Graph $f(x)$.

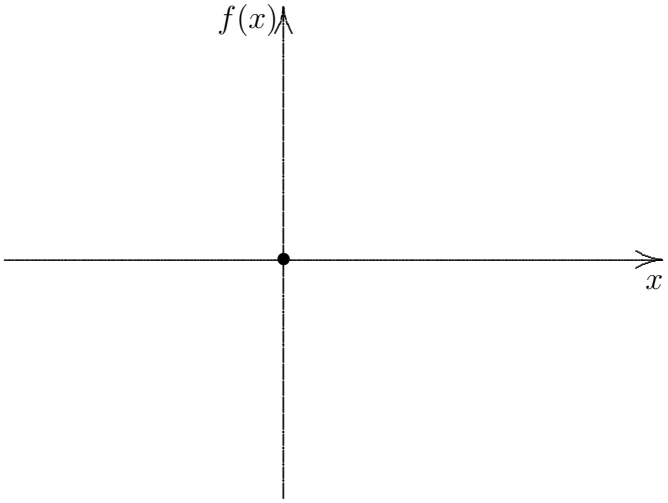

Fig. 2. The graph of $f(x) = 5x + \frac{35}{x}$

Problem 3.(*20 pts.*) Let $f(x) = \frac{x^2}{x^2+1}$. Determine the absolute minimum and the absolute minimum of $f(x)$ on the interval $[-3, 3]$.

Problem 4.(*20 pts.*) From a thin piece of cardboard 10 in. by 10 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?

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Problem 1.(*25 pts.*) Let $f(x) = -x^3 + 3x^2 + 1$.

a) Compute the first derivative $f'(x)$ and find all critical points. Determine the intervals when the function is increasing and when it is decreasing. Use the first derivative test to identify local extrema.

b) Find the second derivative $f''(x)$, all inflection points and intervals where $f(x)$ is concave up and intervals where it is concave down.

c) Graph $f(x)$.

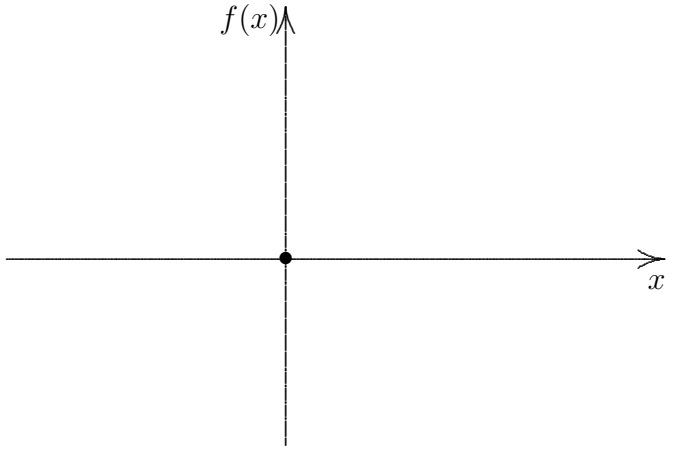

Fig. 1. The graph of $f(x) = -x^3 + 3x^2 + 1$

Problem 2.(*25 pts.*) Find the following derivatives

a)

$$\frac{d}{dx}[(x^2+3)(x^2-3)^{10}] =$$

b)

$$\frac{d}{dx}\left[\frac{x^2+2x}{x+1}\right] =$$

c)

$$\frac{d}{dx}\left[\frac{x^4-4x^2+3}{x}\right] =$$

d)

$$\frac{d}{dx}[(x+3)\sqrt{2x-3}]\ |_6 =$$

e)

$$\frac{d}{dx}\left[(x+1)\sqrt{x^2+3}\right] =$$

Problem 3.(*25 pts.*) An athletic field consists of a rectangular region with semi-circular regions at each end. The perimeter will be used for a 440 yard track. Find the value of x for which the area of the **rectangular region** is as large as possible.

Problem 4. (20 pts.) A one product firm estimates its total const function is $C(x) = x^3 - 6x^2 + 13x + 15$ (in suitable units) and its total revenue function is $R(x) = 28x$. Find the value of x that maximizes the daily profit.

Bonus Problem (10 pts.) A mathematics book is to contain 90 square inches of printed matter per page, with margins of 2 inch along the sides and 3 inches along the top and bottom. Find the dimension of the page that will require the minimum amount of paper.