MATHEMATICS 180

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TOTAL________________

1.(*20 pts.*) Let $f(x) = \frac{2}{x^2+1}$. Consider the point $P = (1,1)$ on the graph of $f(x)$.

a) Find the equation of the line L tangent to $f(x)$ at P.

We have $f'(x) = \frac{-4x}{(x^2+1)^2}$. Hence the slope

$$m = f'(1) = -1$$

and the equation of the line is $y - 1 = -(x-1)$ or $y = -x + 2$.

b) Find points on the graph of $f(x) = \frac{x^3}{3} + 1$ at which the tangent line has slope $m = 1$. (**Sorry for omitting** $f(x)$).

We have $f'(x) = x^2$. The slope $m = 1$ gives the equation

$$f'(x) = x^2 = m = 1.$$

This hast two solutions $x = -1$ and $x = +1$. Hence, there are two tangent lines to the graph of $f(x)$ with slope $m = 1$. They are at points $P_1 = (1, 4/3)$ and $P_2 = (-1, 2/3)$. Thus we have the following tangent lines at these two points

$$y - 4/3 = x - 1,$$

$$y - 2/3 = x + 1.$$

2. (*20 pts.*) Using the power rule, quotient rule, and chain rule compute the derivatives of the following functions:

a) $y(x) = \sqrt{x+1} - \frac{2}{\sqrt{x+1}}$,
$y'(x) = \frac{1}{2\sqrt{x+1}} + \frac{1}{(x+1)^{3/2}}$,

b) $y(x) = \sqrt[7]{x^{10}} + \frac{x^2}{x^5} - \frac{x}{3}$,
$y'(x) = \frac{10}{7}x^{3/7} - 3x^{-4} - \frac{1}{3}$

c) $y(x) = (x^2+2)^{11}$,
$y'(x) = 22x(x^2+2)^{10}$

d) $y(x) = (3x^2+x)(2x+1)^2$,
$y'(x) = (6x+1)(2x+1)^2 + (3x^2+x) \cdot 4(2x+1)$.

e) $y(x) = \frac{x-1}{x^2+1}$.
$y'(x) = \frac{1 \cdot (x^2+1) - 2x(x-1)}{(x^2+1)^2}$.

3.(*20 pts.*) Let $f(x) = x^3 + x$.

a) Compute $f(x+h) - f(x)$,

$$f(x+h) - f(x) = (x+h)^3 + (x+h) - x^3 - x = (x+h)[x^2 + 2xh + h^2 + 1] - x^3 - x =$$

$$[x^3 + 2x^2h + xh^2 + x + x^2h + 2xh^2 + h^3 + h] - x^3 - x = 3x^2h + 3xh^2 + h^3 + h =$$

$$h(3x^2 + 1 + 3xh + h^2).$$

b) Compute the difference quotient $\frac{f(x+h)-f(x)}{h}$ and simplify,

$\frac{f(x+h)-f(x)}{h} = \frac{h(3x^2+1+3xh+h^2)}{h} = 3x^2 + 1 + 3xh + h^2$

c) Compute the limit in b) when $h \to 0$, that is $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}(3x^2 + 1 + 3xh + h^2) = 3x^2 + 1.$$

4.(*20 pts.*) Evaluate the following limits

a)

$$\lim_{x\to 2} \frac{x^3 - 1}{x - 1} = \frac{7}{1} = 7$$

b)

$$\lim_{x\to 1} \frac{1 - \sqrt{x}}{1 - x} = \lim_{x\to 1} \frac{1 - \sqrt{x}}{(1 - \sqrt{x})(1 + \sqrt{x})} = \frac{1}{2}$$

c)

$$\lim_{x\to -\infty} \frac{6x^7 - 5}{2x^6 + 3} = -\infty$$

d)

$$\lim_{x\to -\infty} \frac{4x^3 - 8x + 11}{3x - 3} = +\infty$$

5.(*20 pts.*) A company determines that monthly sales S, in thousands, after t months of marketing a product is given by

$$S(t) = 2t^3 - 40t^2 + 220t + 160.$$

a) Find the monthly sales after 1 month and 4 month.

$$S(1) = 2 - 40 + 220 + 160 = 342.$$

$$S(4) = 128 - 640 + 880 + 160 = 528.$$

b) Find the rate of change $S'(t)$.

$$S'(t) = 6t^2 - 80t + 220.$$

c) Find the rate of change $t = 1$ and $t = 4$.

$$S'(1) = 6 - 80 + 220 = 146.$$

$$S'(4) = 96 - 320 + 220 = -4.$$