MATHEMATICS 180 – Section 005
Midterm Exam #1 – Solutions

1.(*20 pts.*) Let $f(x) = 2x^3$. Consider the point $P = (1, 2)$ on the graph of $f(x)$.

a) Find the equation of the line L tangent to $f(x)$ at P.

$$f'(x) = 6x^2, \qquad f'(1) = 6 = m = \text{slope.}$$

$$y - 2 = 6(x - 1), \qquad \text{or} \qquad y = 6x - 4.$$

b) Find points on the graph of $f(x)$ at which the tangent line has slope $m = 6$. Write down the equations of the tangent lines at these points.

$$f'(x) = 6x^2 = 6 = m \qquad \text{means} \qquad x^2 = 1$$

which has two solutions $x = 1$ and $x = -1$. The first solution gives $P_1 = P = (1, 2)$ and the tangent was derived in part (a). The second solution gives $P_2 = (-1, -2)$ and the equation of the tangent at this point reads $y + 2 = 6(x + 1)$.

2. (*30 pts.*) Compute the following

a) $y(x) = (x+1)^3 - \frac{1}{x^2}$, $\qquad y'(x) = 3(x+1)^2 + \frac{2}{x^3}$

b) $\frac{d}{dx}\left(\sqrt[3]{x^2} \cdot \sqrt[3]{x^4}\right) = \frac{d}{dx}(x^2) = 2x.$

c) $y(x) = (x^2+1)^4$, $\qquad y'(x) = 4(x^2+1)^3(2x) = 8x(x^2+1)^3$

d) $y(x) = \frac{4}{\sqrt{x^2-2x}} = 4(x^2-2x)^{-1/2}$, $\qquad y'(x) = -2(x^2-2x)^{-3/2}(2x-2).$

e) $\frac{d}{dx}(2x^2 - 3) \mid_{x=5} = (4x) \mid_{x=5} = 20$

f) $f(x) = a^2x^2 + 2b^3x + c^4.$

$$\frac{d}{dx}f(x) = 2a^2x + 2b^3, \qquad \frac{d^2}{dx^2}f(x) = 2a^2.$$

3. (*30 pts.*) Compute the following

a) $f(P) = (P+1)^5$, $\qquad \frac{d^2}{dP^2}f(P) = 20(P+1)^3.$

b) $\frac{d^2}{dx^2}(x^3 - x - 1) \mid_{x=2} = (6x) \mid_{x=2} = 12.$

c) $f(x) = (\sqrt{x}+1)^{3/2}.$

$$f'(x) \mid_{x=4} = \frac{3}{4}(\sqrt{x}+1)^{1/2} \cdot x^{-1/2} \mid_{x=4} = \frac{3\sqrt{3}}{8}.$$

d) $f(x) = x^{10}$, $\frac{d^3}{dx^3} f(x) = 720x^7$.

e) $\lim_{x \to 4} \frac{x^2-16}{4+x} = 0/8 = 0$.

f) $\lim_{x \to 2} \sqrt[3]{x^3 + 19} = \sqrt[3]{27} = 3$.

4.(*20 pts.*) Let $f(x) = \frac{3}{x}$.

a) Compute $f(1+h) - f(1) = \frac{3}{1+h} - 3 = \frac{3}{1+h} - \frac{3(1+h)}{1+h} = \frac{-3h}{1+h}$.

b) Compute the difference quotient $\frac{f(1+h)-f(1)}{h} = \frac{-3}{1+h}$.

c) Compute the limit in b) when $h \to 0$, that is

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{-3}{1+h} = -3.$$