Mathematics 327   Sample Exam #1   3 October 2000

Problem 1. Suppose a is rational and b is irrational. Prove that $a+b$ and ab are both irrational.

Problem 2. Suppose $A \cap B = A$. Determine $A \cup B =?$

Problem 3. Determine whether or not each of the binary relations $\mathcal{R}$ is reflexive symmetric, antisymmetric, or transitive:

a) $A = \{1,2\}$, $\mathcal{R} = \{(1,2)\}$.

b) $A = \{1,2,3,4\}$, $\mathcal{R} = \{(1,1),(1,2),(2,1),(3,4)\}$.

c) $A = \mathbb{Z}$, $(a,b) \in \mathcal{R}$ if and only if $ab \geq 0$.

d) $A = \mathbb{R}$, $(a,b) \in \mathcal{R}$ if and only if $a^2 = b^2$.

Problem 4. Let $A = \{1,2,3,4,5,6,7,8,9\}$. For $a,b \in A$ define $a \sim b$ if ab is a perfect square (ie., it is a square of a natural number).

a) List ordered pairs of this relation.

b) For each $a \in A$ find the $\bar{a} = \{x \in A \mid x \sim a\}$.

c) Explain why $\sim$ is an equivalence relation on A.

Problem 5. Draw the Hesse diagram for the following partial order:

$$(\{\{a\},\{a,b\},\{c\},\{a,c\},\{a,b,c\},\{a,b,d\}\},\subseteq).$$

Problem 6. Let $f: A \to A$ be defined by $f(x) = x^2 + 2$.

a) Let $A = \mathbb{Z}$. Determine if f is injective and surjective.

b) Repeat part (a) for $A = \mathbb{R}$.

Problem 7. Let $X = \{a,b\}$ and $Y = \{1,2,3\}$.

a) List all the functions from X to Y.

b) List all the functions from Y to X.

c) List all the injective functions from X to Y.

d) List all the surjective functions from X to Y.

Problem 8. Let $S = \{1,2,3,4,5\}$, and let $f,g,h: S \to S$ be the function defined by

$$f = \{(1,2),(2,1),(3,4),(4,5),(5,3)\},$$

$$g = \{(1,3),(2,5),(3,1),(4,2),(5,4)\},$$
$$h = \{(1,2),(2,2),(3,4),(4,3),(5,1)\}.$$

a) Find $f \circ g$ and $g \circ f$.

b) Find f^{-1}, g^{-1}, and h^{-1} (if they exist).

c) Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1} \neq f^{-1} \circ g^{-1}$.

Problem 9.

a) Find the one-to-one correspondence between the intervals $(1, \infty)$ and $(3, \infty)$.

b) Find the one-to-one correspondence between the intervals $(0, 1)$ and (a, b).

Problem 10.

a) Write the number 1001 in bases $b = 2, 8$.

b) Suppose that in base 12 we use A to denote 10 and B to denote 11. What is the number $1BBA$?