# MATHEMATICS 327

**NAME:** ____________________________________________

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**BONUS PROBLEM.**__________

**TOTAL**________________

**Problem 1.** Let $A, B, C$ be sets. Show that, in general,

$$(A \setminus B) \setminus C \neq A \setminus (B \setminus C).$$

*Hint*: Give any example when this equality is violated.

**Problem 2.** Determine whether or not each of the binary relations $\mathcal{R}$ is reflexive, symmetric, antisymmetric, or transitive:

a) $A = \{1, 2, 3, 4\}$, $\mathcal{R} = \{(1,1), (1,2), (2,1), (3,4), (4,3)\}$.

reflexive:

symmetric:

antisymmetric:

transitive:

b) $A = \mathbb{R}$, $(a, b) \in \mathcal{R}$ if and only if $a - b \leq 3$.

reflexive:

symmetric:

antisymmetric:

transitive:

c) $A = \mathbb{Z}$, $(a, b) \in \mathcal{R}$ if and only if $a + b = 10$.

reflexive:

symmetric:

antisymmetric:

transitive:

d) $A = \mathbb{N}$, $(a, b) \in \mathcal{R}$ if and only if $\frac{a}{b} \in \mathbb{N}$.

reflexive:

symmetric:

antisymmetric:

transitive:

**Problem 3.** Let $S = \{1, 2, 3, 4, 5\}$, and let $f, g, h : S \to S$ be the function defined by

$$f = \{(1,2),(2,1),(3,3),(4,5),(5,4)\},$$
$$g = \{(1,5),(2,3),(3,1),(4,2),(5,4)\},$$
$$h = \{(1,2),(2,2),(3,2),(4,3),(5,1)\}.$$

a) Find $f \circ g$ and $g \circ f$.

b) Find $f^{-1}$, $g^{-1}$, and $h^{-1}$ (if they exist).

c) Find $f^2$, $f^3$, $f^4$. What is $f^n$? (Here $f^2 = f \circ f, f^3 = f \circ f \circ f$, ...)

**Problem 4.** In each case determine if the function is injective (1-1) and/or surjective (onto):

a) $f : \mathbb{N} \to \mathbb{N}, \quad f(n) = 3n$:

b) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = 3x$:

c) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = x$:

d) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = x^4 - x^2$:

**Problem 5.**

a) Write the number 10001 in base $b = 2$.

b) Let $x = (10001)_3$ and $y = (111)_3$. Compute the sum $x + y$ and the product $x \cdot y$ in base $b = 3$.

**BONUS PROBLEM.** Suppose $m, n \in \mathbb{Z}$ and $n^2 + 1 = 2m$. Prove that $m$ is a sum of two squares (i.e., $m = p^2 + q^2$ where $p, q$ are some integers).